Useful information

\[ \sin(x) = \cos(x - 90^\circ) \]  \[ \bar{V} = \bar{Z}_I \]  \[ \bar{s} = \bar{V}I^* = P + jQ \]  \[ \bar{s}_{sp} = \sqrt{3}V_L I_L \angle \theta \]

0 < \theta < 180° (lag) \[ I_L = \sqrt{3}I_\phi \text{ (delta)} \]  \[ \bar{Z}_v = \bar{Z}_\Delta / 3 \]  \[ \mu_0 = 4\pi \cdot 10^{-7} \text{ H/m} \]

−180° < \theta < 0 (lead) \[ V_L = \sqrt{3}V_\phi \text{ (wye)} \]  \[ \Phi = B A \]  \[ \lambda = N\Phi = Li \text{ (if linear)} \]  \[ v = d\lambda/dt \]  \[ k = \frac{M}{\sqrt{L_1L_2}} \]  \[ \text{1 hp = 746 Watts} \]

ABC sequence has A at zero, B at minus 120 degrees, and C at plus 120 degrees

\[ \int_c \mathbf{H} \cdot d\mathbf{l} = \int_s \mathbf{J} \cdot n d\mathbf{a} \]  \[ \int_c \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_s \mathbf{B} \cdot n d\mathbf{a} \]  \[ \mathfrak{R} = \frac{l}{\mu A} \]  \[ \text{MMF} = Ni = \Phi \mathfrak{R} \]

\[ v = \frac{d\lambda}{dt} \]  \[ k = \frac{M}{\sqrt{L_1L_2}} \]  \[ \text{1 hp = 746 Watts} \]

(extra paper at the end)
Problem 1:

Two single-phase loads are connected in parallel to a voltage source through a feeder with impedance $Z_{\text{line}} = 1 + j\sqrt{3} \, \Omega$. Load 1 consumes 1500 W of power at a power factor of 0.8 lagging. Load 2 consumes 1000 VA of power at a power factor of 0.6 lagging. The voltage at the loads is $v_i(t) = \sqrt{2}(120)\cos(377t) \, V$.

Using the given values, find:

a) The total complex power consumed by both loads.

b) The total current supplied by the source.

c) The voltage as a function of time supplied by the source $v_s(t)$.

d) If a capacitor is connected in parallel with the two loads, what power must be supplied to achieve a total power factor of 0.9 lagging?
Problem 1 Solution

\[ \vec{Z}_{\text{line}} \]

\[ \vec{V}_s \]

\[ \vec{V}_1 \]

\[ \vec{V}_2 \]

\[ a) \quad P_1 = 1500 \text{ W} \quad PF_1 = 0.8 \]

\[ P_1 = P_s (PF) \quad S_1 = \frac{P_1}{PF} \quad S_1 = 1875 \text{ VA} \]

\[ S_s = 1500 + j125 \text{ kVA} \quad S_{\text{tot}} = S_1 + S_2 \]

\[ S_2 = 600 + j800 \text{ kVA} \]

\[ S_{\text{tot}} = 2100 + j1925 \text{ VA} = 2498.9^\circ /42.5^\circ \text{ VA} \]

\[ b) \quad \bar{V}_L = 120^\circ \text{ V} \]

\[ S_{\text{tot}} = V_L I_{\text{tot}}^* \]

\[ I_{\text{tot}} = \left( \frac{S_{\text{tot}}}{V_L} \right)^* \quad \bar{I}_{\text{tot}} = 23.74^\circ /42.5^\circ \text{ A} \]

\[ c) \quad \bar{V}_3 = \bar{Z}_{\text{line}} I_{\text{tot}} + V_L \Rightarrow \bar{V}_3 = (2L/4^\circ)(23.74^\circ /42.5^\circ) + 120^\circ \text{ V} \]

\[ \bar{Z}_{\text{line}} = 2L/4^\circ \quad \bar{V}_3 = 47.49^\circ /17.4^\circ + 120^\circ \Rightarrow \bar{V}_3 = 165.9^\circ /4.93^\circ \text{ V} \]

\[ \bar{V}_3 = 165.9^\circ /4.93^\circ \text{ V} \]

\[ V_{\text{rms}} = 14.3^\circ /4.93^\circ \text{ V} \]

\[ d) \quad Q_{\text{tot}} = 1925 \text{ VAR} \Rightarrow Q_C = 1925 \text{ VAR or} -1925 \text{ VAR of capacitance added.} \]
Problem 2. (25 points)

A balanced, symmetrical, Wye-connected, three-phase load consumes a total of 1,000 Watts (3 phase) at a voltage of 208 V (line-line). The line current is 4 Amps and the power factor is lagging.

a) Find the capacitance needed for use in a Delta connection across the load to lower the line current to 3 Amps while the load still consumes the same real power. Assume a 60Hz supply.

\[
\bar{S}_0 = \sqrt{3} \times 208 \times 4 \angle \theta_0 = 1000 + jQ_0 \\
\bar{S}_1 = \sqrt{3} \times 208 \times 3 \angle \theta_1 = 1000 + jQ_1
\]

Need \(1037 - 410 = 627\) Vars 3\(\phi\)

\[
627 = 3 \times \frac{208^2}{x_C} \quad \therefore x_C = \frac{208^2}{627} = 1 \quad \text{pf, pm}\ 
\]

\[
\bar{I} = \frac{1000 + j(1037 - 209)}{3} = 336 + j828 = 1298/\theta_0
\]

b) What would the line current be if these same capacitors were connected in a Wye rather than a Delta?

\[
Q_{new} = 3 \times \frac{120^2}{207} = 209 \text{ Vars 3}\phi
\]

\[
\bar{S}_2 = \sqrt{3} \times 208 \times I \angle \theta_2 = 1000 + j(1037 - 209)
\]

\[
I = 3.6A
\]

c) In the real world, the line that serves the load has a series inductive impedance. If the source voltage is fixed, the load voltage will depend on the load power and the capacitors that are added. Show with a “per-phase” phasor diagram that if enough capacitance is added, the load voltage magnitude can be larger than the source voltage magnitude.
Problem 3. (25 points)

Consider the iron geometry given in the figure below. Assume fringing in the air gap such that \( A_{gap} = 1.1A_{core} \) and assume the following values: \( l_{core} = 10 \) cm, \( l_{gap} = 0.1 \) cm, \( A_{core} = 2 \) cm\(^2\), \( N = 100 \), and \( \mu_r = 1000 \).

(a) Draw the equivalent magnetic circuit.

\[
\begin{align*}
R_C &= \frac{\mu_A}{1000} \frac{0.1}{4\pi \times 10^{-7} \times 0.0002} \\
&= 397.887 \text{ AT/W} \\
R_g &= \frac{0.001}{4\pi \times 10^{-7} \times 0.0002} \\
&= 3.61 \times 10^6 \text{ AT/W} \\
R_2 &= \frac{(0.1 - 0.001)}{1000 \times 4\pi \times 10^{-7} \times 0.0002} \\
&= 393908 \text{ AT/W} \\
\end{align*}
\]

(b) Find the inductance of the coil.

\[
\begin{align*}
\Phi_1 &= 100i \\
L &= \frac{N\Phi}{\frac{1}{R_{eq}}} \\
&= \frac{N^2}{R_{eq}} \\
&= 0.001473 \text{ H} \\
&= 4.73 \text{ mH}
\end{align*}
\]

Continued on the next page
(c) Find the current needed to generate a flux in the middle leg of $5 \times 10^{-6}$ Wb.

\[
(\Phi_1 - \Phi_2) = 5 \times 10^{-6} \Rightarrow \Phi_1 = 5 \times 10^{-6} + \Phi_2 \quad \Phi_2 = \frac{MMF}{3R_L}
\]

\[
5 \times 10^{-6} \times (R_L + R_g) = MMF = 20.0553
\]

\[
R_L = 0.000017
\]

\[
\Phi_1 = 5 \times 10^{-6} + 0.000017 = 0.000022
\]

Loop 1: \[-N_i + 3R_L \Phi_1 + MMF = 0\]

\[
\frac{A_i}{A} = \frac{(3R_L \Phi_1 + 20.0553)/110}{0.463A}
\]

(d) Find the flux density (Wb/m$^2$) in the right leg corresponding to the values given in part c.

\[
\Phi_2 = 0.000017
\]

\[
B_2 = \frac{\Phi_2}{A_2} = \frac{0.000017}{0.0002} = 0.085 \text{ Wb/m}^2
\]
Problem 4: (25 points)

One type of magnetic actuator consists of a moving piston and two coils, as shown above. Coil 1 acts as a constant MMF source with constant current $i_1$ whose direction is given. Coil 2 can produce an MMF that is used to either open or close the actuator through current $i_2$. The number of turns for each coil is given as $N_1$ and $N_2$ respectively. There is a constant air gap $g$ between Coil 2 and the moving piston, whose position is given by $x$. The areas that the magnetic field acts through in the air gap and piston location are $A_g$ and $A_x$ respectively. Assume that the iron core and moving piston have infinite permeability, and that the magnetic flux acts all the way around a counter-clockwise loop.

Using the current directions and polarity definitions given:

a) Find the dot convention for the given coils.

b) Draw the magnetic equivalent circuit for the actuator.

c) The self-inductance of coil 2, $L_2$, and the mutual inductance $M$ in terms of $x, g, N_1, N_2, A_g$, and $A_x$.

d) Qualitatively, what happens to $L_2$ as the actuator opens ($x$ increases from 0 to 1): increase, decrease, or stay the same?

e) Find $i_2$ needed for zero flux through the iron.

Extra paper on the next page
Problem Solution

a)

b)

\[ N_1 \omega_1 - N_2 \omega_2 = \phi (R_g + R_x) \]
\[ N_1 \omega_1 - N_2 \omega_2 = \phi (R_g + R_x) \]

\[ \phi = \frac{N_1 \omega_1 - N_2 \omega_2}{R_g + R_x} \]
\[ \phi = \frac{N_1 \omega_1 - N_2 \omega_2}{R_g + R_x} \]

\[ \lambda_2 = \frac{N_1 \omega_2}{N_2} \Rightarrow \lambda_2 = \frac{N_1}{N_2} \left( \frac{X_2}{X_1} \right) \]
\[ \lambda_2 = \frac{N_1 \omega_2}{N_2} \Rightarrow \lambda_2 = \frac{N_1}{N_2} \left( \frac{X_2}{X_1} \right) \]

\[ L_2 = \frac{N_2^2}{L_1} \Rightarrow L_2 = \frac{N_1 N_2^2}{L_1} \]
\[ L_2 = \frac{N_2^2}{L_1} \Rightarrow L_2 = \frac{N_1 N_2^2}{L_1} \]

\[ R_g = \frac{g}{\mu A_2} \]
\[ R_x = \frac{x}{\mu A_2} \]

\[ R_a = \frac{g^2}{\mu A_2} + \frac{x^2}{\mu A_2} \]

\[ R_a = \frac{g^2}{\mu A_2} + \frac{x^2}{\mu A_2} \]

\[ L_2 = \frac{N_2^2}{L_1} \Rightarrow L_2 = \frac{N_1 N_2^2}{L_1} \]
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\[ \text{As } x 	ext{ increases, } L_2 \text{ decreases} \]
\[ \text{As } x 	ext{ increases, } L_2 \text{ decreases} \]

\[ \phi = \frac{N_1 \omega_1 - N_2 \omega_2}{R_x + R_g} \]
\[ \phi = \frac{N_1 \omega_1 - N_2 \omega_2}{R_x + R_g} \]

\[ N_1 \omega_1 - N_2 \omega_2 = 0 \Rightarrow i_2 = \left( \frac{V_1}{R_x} \right) i_1 \]
\[ N_1 \omega_1 - N_2 \omega_2 = 0 \Rightarrow i_2 = \left( \frac{V_1}{R_x} \right) i_1 \]