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TOTAL: 

Three sheets provided
Problem 1 (10 pts. – no partial credit)

a) If \( v(t) = 200 \cos(377t - 10^\circ) \) and \( i(t) = 10 \sin(377t + 125^\circ) \), find the complex
\[ P + jQ, \text{ PF (specifically lead or lag)} \]
\[ P + jQ = 707 - j707 \quad \text{P.F. = 0.707} \quad \text{Lead or Lag} \]
\[ \vec{V} = \frac{200}{\sqrt{2}} L_{-10^\circ}, \quad \vec{I} = \frac{10}{\sqrt{2}} L_{35^\circ} \]
\[ \vec{S} = 1000 L_{-45^\circ} \]

b) Two single phase loads in parallel, 10 kVA at 0.8 PF lag and 16 kW at 0.8 PF lead are supplied by a source \( \vec{V} = 240 L_{0^\circ} \). Find the total current (magnitude) supplied by the source and the combined PF (specify lead/lag).
\[ I = 10.3 \quad \text{P.F. = 0.97} \quad \text{Lead or Lag} \]
\[ \vec{S} = 10 \frac{kVA}{L_{37^\circ}} + 16 \frac{kW}{L_{-37^\circ}} = 7.986 + j6018 \]
\[ = 23.959 - j5618 \]
\[ \vec{S} = 24703 L_{-14^\circ} = 240 \quad \text{P.F. = 0.97} \]
\[ \text{P.F. = \cos(-14^\circ) = 0.97} \]

\[ I = \frac{24703}{240} = 103 \]

\( 3 \times 1118 = \sqrt{3} \times 4180 \) \( \text{I} \)
\[ \text{I} = 4,03 \]

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Problem 2 (10 pts. – no partial credit)

Given the following device

\[ H_g = N i \]

\[ \mu_{res} = \text{cross section area } A \]

Find the following in terms of \( \mu_r, N, A, g, i \)

a) \( H_{ex} \) (directed down) = \( \frac{N i}{g} \)

b) \( B_{ex} \) (directed down) = \( \frac{\mu_0 N i}{g} \)

c) \( B_{sin} \) (directed clockwise) = \( \frac{\mu_0 N i}{g} \)

d) \( \phi \) (directed clockwise) = \( \frac{\mu_0 A N i}{g} \)

e) \( \lambda \) (where \( \lambda = \frac{d\phi}{dt} \)) = \( \frac{\mu_0 A N^2 i}{g} \)
Problem 3 (10 pts. – no partial credit)

a) A coil of 500 turns is wound on an iron core whose reluctance $\mathcal{R} = 4.6 \times 10^6$ A/uW. The inductance of the coil is \( 55 \mu \text{H} \).

\[ R \Phi = NI \quad \Rightarrow \quad \frac{V}{R} = \frac{NI}{R} = L i \]

b) Two coils which are coupled have self-inductances of 10 and 20 mH respectively and a coupling coefficient of 0.9. The mutual inductance is \( 12.7 \text{mH} \).

\[ m = \frac{Q}{\sqrt{L_1 L_2}} = \frac{9}{\sqrt{10 \times 20}} = 12.7 \text{mH} \]

c) Input resistance at “ab” is 100Ω (R > 0). The value of R is 9.

\[ 100 = \frac{V}{R + i_b} \quad \Rightarrow \quad R = 0 \]

d) Write the two loop equations for the circuits shown.

\[
\text{Left:} \quad V = (i_1 + i_2) R_0 + L_1 \frac{di_1}{dt} + i_1 R_1 - m \frac{di_2}{dt} \\
\text{Right:} \quad 0 = R_2 i_2 + L_2 \frac{di_2}{dt} - m \frac{di_1}{dt} - R_1 i_1 - L_1 \frac{di_1}{dt} + m \frac{di_2}{dt}
\]
Problem 4 (10 pts. – no partial credit)

a) An electromechanical device has the following flux-linkage versus current relationship:

\[ \lambda = \frac{-0.04}{i} \cdot \frac{1}{0.02 + x} \]

The system starts at point a \((i = 5A, x = 0)\) and moves to point b \((i = 5A, x = 0.02m)\) along a constant current path. Find the following:

\[ W_m = 12.5 \text{ J} \]

\[ EFE = -25 \text{ J} \]

\[ EFM = 12.5 \text{ J} \]

b) An R-L circuit has the following differential equation:

\[ \frac{di}{dt} = -2i + 12 \quad i(0) = 0 \]

Use Euler's method with a step size of 0.01 sec to estimate the current at 0.01 and 0.02 seconds.

\[ i(0.01) = 0.1 + 0.01(0 + 12) = 0.12 \text{ A} \]

\[ i(0.02) = 0.12 + 0.01(0.12 + 12) = 0.2376 \text{ A} \]
Problem 5 (10 pts. – no partial credit)

A nonlinear dynamic model of a system is:

\[
\frac{dx}{dt} + x^3 - 16 = 0
\]

a) The two equilibrium points for this system are:

\[x_1 = 4\]

\[x_2 = -4\]

b) The linearized model valid for either \(x_n\) is

\[
\frac{d\Delta x}{dt} = -2x_e \Delta x
\]

c) Is \(x_1\) a stable or unstable (circle one) equilibrium point?

d) Is \(x_2\) a stable or unstable (circle one) equilibrium point?
Problem 6 (10 pts. – no partial credit)

a) A two phase synchronous machine (round rotor) is shown below

Write the 3x3 inductance matrix for this machine in terms of $L_A$, $L_r$, $M$ and $\theta$

$$
\begin{align*}
\begin{bmatrix}
    L_A & M 
    M & L_r 
    M & M
\end{bmatrix}
\end{align*}
$$

b) A single coil is located on the stator of a salient-pole machine as shown below

When $\theta = 0$, the self inductance is a maximum value $L_A$

When $\theta = \frac{\pi}{2}$, the self inductance is a minimum value $L_B$

Write a general form of the self inductance of the coil in terms $L_A$, $L_B$, and $\theta$

$$
L = L_0 + L_1 \cos 2\theta
$$

$$
L_A = L_0 + L_1 \Rightarrow L_0 = \frac{L_A - L_B}{2}
$$

$$
L_B = L_0 - L_1 \Rightarrow L_1 = \frac{L_A - L_B}{2}
$$

$$
L = \frac{L_A + L_B}{2} + \left(\frac{L_A - L_B}{2}\right) \cos 2\theta
$$
Problem 7 (10 pts. – no partial credit)

A 3-phase, 4-pole, 60 Hz induction motor is running at a slip of 0.05

a) The speed of the motor is \( 1710 \) RPM

b) The speed of the motor is \( 179 \) mechanical radians per second

c) The frequency of the rotor currents is \( 3 \) Hz

d) The rotor copper losses are \( 5.26 \) % of the output power (Pm)

e) The air gap power is \( 105.26 \) % of the output power (Pm)

\[
I^2 R \quad vs \quad I^2 R \left( \frac{45}{25} \right) \\
I^2 R \quad vs \quad I^2 R \left( \frac{15}{10} \right)
\]
Problem 8 (30 pts.)

A 100 kVA, 2300/230 V, single phase 60 Hz transformer has the following parameters.
(Side 1 is HV and side 2 is LV)

\[
R_1 = 0.3 \Omega, \quad R_2 = 0.003 \Omega, \quad R_{SL} = \infty \\
X_{HL} = 0.65 \Omega, \quad X_{HS} = 0.0065 \Omega, \quad X_{SL} = \infty
\]

Transformer delivers 75 kW at 230 V at 0.85 PF logging

1) Find

(a) input voltage
(b) input current
(c) input power
(d) efficiency
(e) voltage regulation

2) Suppose the transformer is accidentally short circuited at the load terminals, what is the input current assuming input voltage does not change?

\[
\begin{align*}
\text{Output} & = (0.6 + j1.3)(38.36 L^{-32^\circ}) + 2300 L^0 \\
& = 54.9 L^{33^\circ} + 2300 L^0 = 2346 L^{33^\circ}
\end{align*}
\]

\[
\begin{align*}
\text{(a)} & \quad V_L = \left(0.6 + j1.3\right)\left(38.36 L^{-32^\circ}\right) + 2300 L^0 \\
& = 54.9 L^{33^\circ} + 2300 L^0 = 2346 L^{33^\circ}
\end{align*}
\]

\[
\begin{align*}
\text{(b)} & \quad I_L = 38.36 L^{-32^\circ} \\
& = 54.9 L^{33^\circ} + 2300 L^0 = 2346 L^{33^\circ}
\end{align*}
\]

\[
\begin{align*}
\text{(c)} & \quad \delta = 2346 L^{33^\circ} \\
& = 89.9\% \text{ LL} \times 38.36 L^{33^\circ}
\end{align*}
\]

\[
\begin{align*}
\text{(d)} & \quad \eta = \frac{75000}{75474} \times 100 = 99\%
\end{align*}
\]

\[
\begin{align*}
\text{(e)} & \quad \text{LR} = \frac{2346 - 2300 \times 100}{2300} = 2\%
\end{align*}
\]

\[
\begin{align*}
\text{2) } \quad I_{SC} = \frac{2346 L^{33^\circ}}{0.6 + j1.3} = 1638 \text{ AMPs}
\end{align*}
\]
Problem 9 (30 pts)

A 230 Volt (line to line), 2-pole, 3-phase, 60Hz, balanced, symmetrical, round-rotor synchronous machine has negligible armature (stator) resistance. Two tests were performed on the machine as follows:

Open-circuit test:
- $I_b = 0$ (no line current)
- $V_a = 120V$ (rated open circuit voltage line to neutral)
- $I_f = 4$ Amps (field current)

Short-circuit test:
- $I_b = 5$ Amps (rated line current)
- $V_a = 0$ (shorted stator terminals)
- $I_f = 2$ Amps (field current)

a) Compute $M$, where the magnitude of the internal voltage $E_{int}$ is equal to $\frac{E_{ph}M}{\sqrt{2}}$.

$$\frac{\frac{2\pi 40 \times 4}{\sqrt{2}}}{\sqrt{2}} = 12.0$$

$$M = 0.113$$

b) Compute the synchronous reactance $X_s$.

$$\frac{3.77 \times 1.13 \times 2}{\sqrt{2}} = 5 \times X_s$$

$$X_s = 12$$

If you cannot solve a) for $M$ and $X_s$, then use $M=0.1$ and $X_s = 10$ for the remainder of this problem.

c) Compute the line current $I_a$ and field current $I_f$ for the following three cases where the machine is loaded as a generator to 2,300 Watts (3-phase) with the terminal voltage fixed at 120 Volts (line to neutral):

- minimum field current ($\delta = 90$ degrees)
- unity power factor ($Q = 0$)
- delivers 1,000 VARS (3-phase) to the load

(USE BLANK PAGE PROVIDED AFTER THIS PAGE)
(1) \( S = 90^\circ \) 
\[ 2300 = \frac{3 \times 120 \times 3372 \times 113 \times I_c \sqrt{3}}{12} \]
\[ I_c = 2.6 A \]
\[ \sin 90^\circ = \frac{120 - \sqrt{3}}{12} \]
\[ I_a = 11.8 A \]

(2) \( 3 \times 120 I_a = 2300 \)
\[ I_a = 6.4 A \]
\[ \frac{3372 \times 113 \times I_c}{\sqrt{3}} \]
\[ I_c = 6.410 \]
\[ j12 + 120 = 142 J35^\circ \]
\[ I_a = 4.73 \]

(3) \( 2700 + j1000 = 3 \times 120 L \)
\[ I_a \]
\[ I_a = 7.1 - 23.5^\circ \]
\[ \frac{3372 \times 113 \times I_c}{\sqrt{3}} \]
\[ I_c = 7.1 - 23.5 \times j12 + 120 L \]
\[ = 84 \]
\[ 165 + 120 L = 153 + j77 \]
\[ = 173 L \]
\[ I_r = 5.7 A \]
Problem 10 (30 pts)

A 230 Volt (line to line), 6-pole, 3-phase, 60Hz, balanced, symmetrical, round-rotor induction machine has negligible stator copper loss, negligible core loss, and negligible stator leakage reactance. The magnetizing reactance as seen on the stator side is 40 Ohms and the rotor leakage reactance as seen on the stator side is 2 Ohms. The full-load (Pm = 3,700Watts 3-phase) speed is 1050 RPM.

a) What are the rotor copper losses?

\[ P_{\text{cl}} = 3.700 \times \frac{3}{8} \times \left( \frac{1 - 12\%}{1 - 15\%} \right) = 7P_{\text{cl}} \]

\[ P_{\text{cl}} = \frac{3.700}{2} = 529 \text{ W} \]

b) What is the full-load torque in Newton-Meters?

\[ T_{\text{fl}} = \frac{3.700}{1050 \times \frac{30}{60}} = 33.6 \text{ Nm} \]

c) What is the maximum torque that this motor can deliver?

\[ T_{\text{mke}} = \frac{b \times 2}{2} \times \frac{(2.30 \frac{\text{V}}{\sqrt{s}})^2}{2\pi \times 60 \times 2} = 105 \text{ Nm} \]