1. (30 points total)

A delta connected load with each \( Z = 8 \angle 30^\circ \Omega \) is supplied from a balanced three phase wye connected generator through lines having an impedance of \( 0.25 \angle 30^\circ \Omega \). Line-to-line voltage at the load is 208 volts (for the angle, assume that at the load \( V_{ab} = 208 \angle 0^\circ \) volts).

(10) a. Calculate the three load phase currents (magnitude and angle).

(10) b. Calculate the magnitude of the line-to-line voltage at the generator.

(10) c. Calculate the power factor at the generator (be sure to indicate leading or lagging).

\[
I_{ab} = \frac{V_{ab}}{Z} = \frac{208 L_0}{8 L 30} = 26 L - 30^\circ \text{ A}
\]

\[
I_{bc} = 26 L - 150^\circ \text{ A}
\]

\[
I_{ca} = 26 L - 270^\circ \text{ A} \quad \text{or} \quad 26 L + 90^\circ \text{ A}
\]

b) Use per phase analysis

\[
V_{an'} = 120 L - 30^\circ + 45 L - 60^\circ = 0.25 L 30^\circ
\]

\[
V_{a'b'} = 131.25 L - 30^\circ
\]

\[
\text{PF} = \cos 30^\circ = 0.866 \text{ lag}
\]
2. (35 points total – 7 points for each part)

A 240/120-V, 6.0-kVA, 60-Hz, single-phase transformer is used to supply a 6.0 kVA resistive load (i.e., the load has a power factor of 1.0) at rated voltage of 120 V.

a) If you assume the transformer is ideal, what would be the magnitude of the expected primary side (240 Volt side) current?

b) Again, if you assume the transformer is ideal, what is the apparent impedance of the load viewed from the primary side of the transformer?

Next, the transformer is tested to determine its parameters. Interestingly, during the open circuit, \( I_{oc} = 0 \) and \( P_{oc} = 0 \). During the short circuit test, in which a voltage is applied to the primary (high side) in order to get rated current to flow in the secondary, \( V_{sc} = 12.5 \) Volts and \( P_{sc} = 187.5 \) W.

c) What are \( R_{eq} \) and \( X_{eq} \) (referred to the primary) ?

d) What are \( X_m \) and \( R_c \) (referred to the primary) ?

e) Using the model values determined from the test data, what voltage must be applied to the primary in order to supply the 6.0 kVA resistive load at a rated voltage of 120 V?

\[
9) \quad \frac{6000}{120} = 50 \text{A} = I_s \quad \Rightarrow \quad I_p = 25 \text{A} \\
\]

\[
b) \quad \frac{240V}{125 \text{A}} = 1.92 \text{Ω} \\
\]

\[
<) \quad \text{In sc test } I_p = 25 \text{A} \Rightarrow 25^2 ; R_{eq} = 187.5 \text{W} \quad \Rightarrow \quad R_{eq} = 0.3 \Omega \\
12.5V / 25 \text{A} = 0.5 \Omega \quad \Rightarrow \quad 0.5^2 = R_{eq}^2 + X_{eq}^2 \quad \Rightarrow \quad X_{eq} = 0.4 \Omega \\
\]

d) \( X_m \div R_c = \infty \)

e) \( V_s = 340 + 35 \angle(63 + 94) \quad \Rightarrow \quad V_s = 247.7 + j10 \quad \Rightarrow \quad 247.7 \text{V} \)
(7 pts) a)  Draw the circuit model for the real transformer discussed in class, and explain the physical significances of each component in the model.

\[ V_p \]

\[ R_c \]

\[ jX_m \]

\[ \Phi_{eq} \]

\[ V_s \]

\[ R_c - \text{models core losses} \]

\[ jX_m - \text{models need for magnetizing current to maintain core flux} \]

\[ R_{eq} - \text{models resistance in winding} \]

\[ jX_{eq} - \text{models leakage flux} \]

(7 pts) b)  A 200 turn coil is wrapped around a toroid that has an average radius of 10 cm, a cross-sectional area of 50 cm² and a relative permeability of \( \mu_r = 500 \). Assuming that all flux is within the core, how much current must flow in the coil to generate a flux (in the core) of 0.005 Weber?

\[
\Phi = B \cdot A = \mu_0 \mu_r H \cdot A = \mu_0 \mu_r \frac{V_i}{\partial \Phi / \partial t} \cdot A
\]

\[
i = \frac{\Phi \cdot 2\pi R}{\mu_0 \mu_r N \cdot A} = \frac{0.005 \cdot 2\pi \cdot 0.1}{4\pi \cdot 10^{-7} \cdot 500 \cdot 200 \cdot 50 \cdot 10^{-4}} = 5 \text{ A}
\]

(7 pts) c)  Put dot markings on the coils
Three loads are connected in parallel as follows: Load 1 consumes 8 kW at a lagging power factor of 0.8, load 2 consumes 10 kVA at a lagging power factor of 0.9, load 3 consumes $4 + 3j$ kVA. How much capacitance (in kvar) must be connected in parallel with these three loads to correct the total power factor to 0.9 leading?

\[
\begin{align*}
\text{Total load} &= 8 + j6 + 9 + j4.36 + 4 + 3j \text{ kVA} \\
&= 21 + j13.36 \text{ kVA} \\
Q_{\text{desired}} &= \frac{21}{0.9} \sqrt{1 - 0.9^2} = -10.17 \text{ kV}a \\
Q_{\text{cap}} &= 13.36 - (-10.17) = 23.53 \text{ kV}a
\end{align*}
\]

Write the left and the right loop equations for the below circuit in terms of $i_1$, $i_2$ and the circuit parameters. Evaluate both loops in the clockwise direction. Note that $L_2$ is coupled to both $L_1$ and $L_3$.

\[
\begin{align*}
V &= L_1 \frac{di_1}{dt} + L_3 \frac{d(i_1 - i_2)}{dt} - m_1 \frac{di_1}{dt} - m_2 \frac{di_2}{dt} \\
0 &= L_2 \frac{di_2}{dt} + R i_b + L_3 \frac{d(i_2 - i_1)}{dt} + m_2 \frac{di_2}{dt} + m_3 \frac{di_3 - i_1}{dt} \\
&\quad - m_1 \frac{di_1}{dt}
\end{align*}
\]