USEFUL INFORMATION

\[
\sin x = \cos(x - 90^\circ) \quad \overline{E}_y = \frac{1}{3} \overline{E}_A
\]

\[
\overline{S}_{\overline{3} \overline{b}} = \sqrt{3} V_L I_L L_0 \quad I_L = \sqrt{3} I_\phi \ (dc\text{ in})
\]

\[
\oint c \overline{H} \cdot dl = \oint c \overline{B} \cdot dl \quad \oint c \overline{E} \cdot dl = -\frac{d}{dt} \oint c \overline{B} \cdot dl
\]

\[
\oint c \overline{B} \cdot ds = 0 \quad R = \frac{l}{\mu A} \quad \text{mmf} = NI = \phi R
\]

\[
\mathcal{A} = N\phi = LI \quad \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \quad k = \frac{M}{VL_1 L_2}
\]
Problem 1 (42 pts.) (No partial credit - 6 points each)

a) A single-phase load has a voltage of $157 \cos (377t + 15^\circ)$ Volts with a current into the positive terminal of $12 \sin (377t + 70^\circ)$ Amps.

\[
\begin{align*}
V &= 111.03 \angle 15^\circ \\
I &= 8.49 \angle -20^\circ \\
S &= 942 \angle 35^\circ
\end{align*}
\]

The real power absorbed by this load is \(772\) Watts.

b) Two loads in parallel consume the complex powers of $100+j100$ kVA and $50-j20$ kVA.

\[
S = 150k + j380k = 170k \angle 28^\circ
\]

The power factor of the total load is \(0.88\) (specify lead or circle one)

c) A single-phase load resistor of 20 Ohms is being served through a line with an inductive reactance of value \(X\). The source voltage magnitude is 122 Volts and the load voltage magnitude is 120 Volts.

\[
\begin{align*}
122 \angle 0^\circ + jX &= 6 \angle 0^\circ + 120 \\
122 \cos \theta &= 120 \\
\theta &= 10.4^\circ \\
122 \sin \theta &= 22 = 6X
\end{align*}
\]

The value of \(X\) is \(3.67\) Ohms.

d) A 3 phase, delta connected load has a line to line voltage of 480 V. The complex power per phase is $2,000+j700$ VA.

\[
S_3 = 6000+j2100 = 6357 \angle 19^\circ = \sqrt{3} \times 480 \angle 0^\circ
\]

The magnitude of the line current is \(7.65\) A.
e) A coil of 300 turns is wound on an iron core whose cross sectional area is 0.002 square meters. The applied voltage is $120\sqrt{2} \cos(2\pi 60t)$ Volts.

\[ 120\sqrt{2} (\cos(2\pi 60t)) = 300 \frac{d\phi}{dt} \quad \phi = \frac{120\sqrt{2}}{300} \sin 2\pi 60t \]

\[ B = \frac{\phi}{A} = \frac{120\sqrt{2}}{0.002 \times 300 \times 2\pi 60} \sin 2\pi 60t \]

The peak value of the magnetic flux density is $0.75$ Tesla.

f) Find the turns ratio “a” to maximize the power absorbed by the 5Ω load in the impedance matching circuit shown below:

\[ a^2 \sqrt{2} = \sqrt{10^2 + 10^2} \]

The turns ratio “a” should be $1.68$.

g) Put the polarity dot markings on the two coils shown below.
Problem 2 (29 pts)

The following three-phase balanced loads are connected in parallel across a three-phase wye-connected, 60 Hz source of 4,160 V (line to line).

Load #1 120 kVA at 0.8 PF lag (Wye connected)
Load #2 180 kW at 0.7 PF lag (Wye connected)
Load #3 13 Amps phase current, unity power factor (Delta connected)

a) Find the total complex power consumed by the three loads
b) Find the total source line current (magnitude).

c) Find the CAPACITANCE needed per phase (for a delta connection) so that the overall power factor is 0.95 lag.

d) Find the new source line current with the 3-phase bank of capacitors installed.

\[
\begin{align*}
\bar{S}_{\text{total}} &= 120 \text{kV}A \left[ 327^\circ \right] + \frac{180 \text{kW}}{0.7} \left[ 46^\circ \right] + 3 \times 13 \times 4160 \text{L}
\\
&\approx 95.8 \text{kV}A + j 22.2 \text{kV}A
\\
&+ 180 \text{kW} + j 185 \text{kW}
\\
&+ 162 \text{kW}
\\
&= 438 \text{kVA} + j 257 \text{kVA}
\\
&\approx 70 \text{Amps}
\end{align*}
\]

\[
\begin{align*}
\sqrt{(438 \text{kVA})^2 + (257 \text{kVA})^2} &= \sqrt{3} \times 4160 \text{Il}
\\
\text{Il} &= 70 \text{Amps}
\end{align*}
\]

\[
\begin{align*}
\bar{S}_{\text{new}} &= 461 \text{kVA}
\\
\Omega_{\text{new}} &= 142 \text{kVAr}
\\
\Omega_{\text{del}} &= (257 - 142) \text{k} = 115 \text{k} = 3 \times 4160^2 \times 2\pi \times 60 \text{C}
\\
\text{C} &= 5.88 \text{µF}
\\
\bar{I}_{\text{new}} &= 461 \text{kVA} \times 4160 \text{Il}
\\
\text{Il}_{\text{new}} &= 64 \text{Amps}
\end{align*}
\]
Problem 3 (29 pts)

\[ \mu = 1000 \mu_0 \]

N_1 = 50 turns
N_2 = 25 turns

A = 10 cm² everywhere

Neglect leakage flux, coil resistance and core losses.

a) Find the coil self inductances (L_1 and L_2) plus the mutual inductance (M)

b) Find the flux density (B) in each vertical leg if the current \( i_1 = 5 \) Amps (DC), and the current \( i_2 = -10 \) Amps (DC).

\[ R_1 = R_2 = \frac{0.15}{1000 \times \pi \times (10^7) \times 0.01} \]

\[ = 119,366 \]

\[ R_3 = \frac{R_1}{3} = 39,789 \]

\[ -50i_1 + 119,366 \phi_1 + 39,789 (\phi_1 - \phi_2) = 0 \]

\[ -25i_2 + 119,366 \phi_2 + 39,789 (\phi_2 - \phi_1) = 0 \]

\[
\begin{bmatrix}
159,155 & -39,789 \\
-39,789 & 159,155
\end{bmatrix}
\begin{bmatrix}
\phi_1 \\
\phi_2
\end{bmatrix}
=
\begin{bmatrix}
50i_1 \\
25i_2
\end{bmatrix}
\]
\[
\begin{bmatrix}
\phi_1 \\
\phi_2
\end{bmatrix} = \begin{bmatrix}
159,155 \\
39,789
\end{bmatrix} \begin{bmatrix}
39,789 \\
159,155
\end{bmatrix} \begin{bmatrix}
5041 \\
2512
\end{bmatrix} \\
\frac{2.375 \times 10^{10}}{}
\]

\[
\phi_1 = 335 \times 10^{-6} i_1 + 41.8 \times 10^{-6} i_2 \\
\phi_2 = 83.8 \times 10^{-6} i_1 + 167 \times 10^{-6} i_2
\]

\[
\begin{align*}
\lambda_1 &= 5041 \phi_1 = 0.01675 i_1 + 0.0021 i_2 \\
\lambda_2 &= 25 \phi_2 = 0.0021 i_1 + 0.004175 i_2
\end{align*}
\]

\[
\begin{align*}
L_1 &= 0.01675 \text{ H} \\
L_2 &= 0.004175 \text{ H} \\
m &= 0.0021 \text{ H}
\end{align*}
\]

b) \[
\begin{align*}
\beta_1 &= \phi_1 \frac{A}{A} = \frac{335 \times 10^{-6} \times 5}{0.01} - \frac{41.8 \times 10^{-6} \times 10}{0.01} = 1.258 \text{ T} \\
\beta_2 &= \phi_2 \frac{A}{A} = \frac{83.8 \times 10^{-6} \times 5}{0.01} - \frac{167 \times 10^{-6} \times 10}{0.01} = -1.257 \text{ T}
\end{align*}
\]

\[
\beta_3 = \beta_1 - \beta_2 = 2.5 \text{ T}
\]