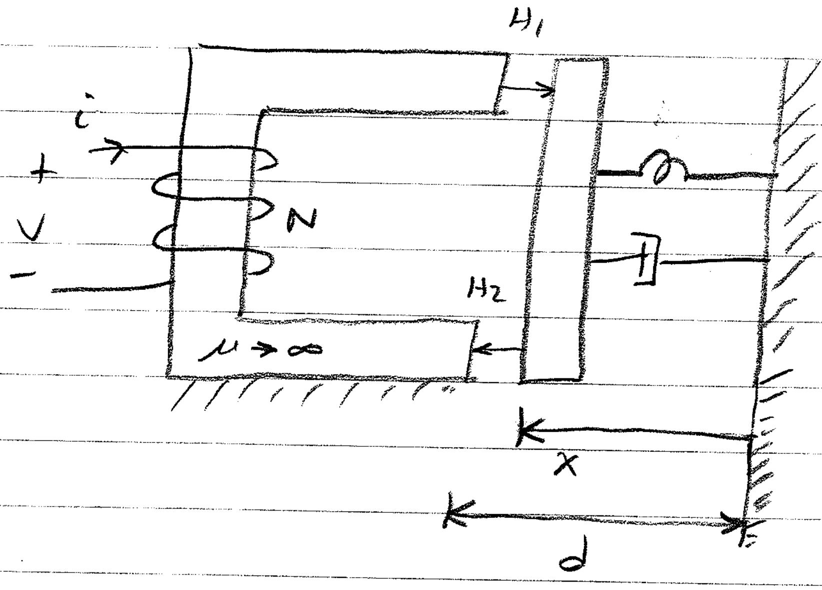
#16 night Exam I

#17 Cancel

Chapter 4 Electromechanics

Translational system



Amperes law $H_1(d-x) + H_2(d-x) = Ni$

Gauss's law $-\mu_0 H_1 A + \mu_0 H_2 A = 0$

Solve: $H_1 = H_2 = \dfrac{Ni}{2(d-x)}$

$$\phi = \mu_0 H_1 A = \frac{\mu_0 A N}{2(d-x)} i$$

$$\lambda = N\phi = \frac{\mu_0 A N^2}{2(d-x)} i = L(x)\, i$$

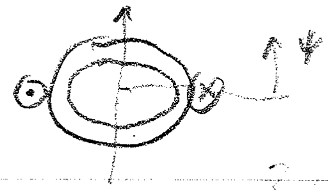
Faraday's law

$$-v = -\frac{d\lambda}{dt}$$

$$v = \frac{d}{dt}\left(\frac{\mu_0 A N^2}{2(d-x)}\, i\right)$$

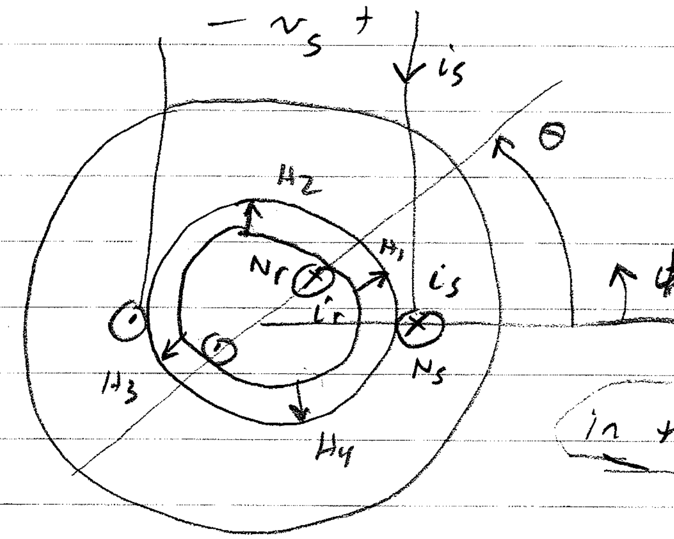
ν

$$v = \underbrace{\frac{\mu_0 A N^2}{2(d-x)}\frac{di}{dt}}_{\text{Transformer voltage}} + \underbrace{\frac{\mu_0 A N^2 i}{2(d-x)^2}\frac{dx}{dt}}_{\text{Speed voltage}}$$

Start with 

#19

Rotational System



depth l

$g << R$

$\mu \to \infty$

mean Radius R

(x) is also + for v

in this figure $0 < \theta < \pi$

Only 4 different H values (apply Ampere to show)

Assume H is radial out from rotor to stator

$H(\phi = 70°) = H_1$ etc.

Ampere
$$H_1 g - H_4 g = N_s i_s$$
$$H_1 g - H_3 g = N_s i_s - N_r i_r$$
$$H_1 g - H_2 g = -N_r i_r$$

Stop 2007

$$\cancel{\mu_0} H_1 \theta \cancel{R l} + \cancel{\mu_0} H_2 (\pi - \theta) \cancel{R l} + \cancel{\mu_0} H_3 \theta \cancel{R l} + \cancel{\mu_0} H_4 (\pi - \theta) \cancel{R l} = 0$$

$$H_1 \theta + \left(H_1 + \frac{N_r i_r}{g}\right)(\pi - \theta) + \left(H_1 + \frac{N_r i_r - N_s i_s}{g}\right)\theta + \left(H_1 - \frac{N_s i_s}{g}\right)(\pi - \theta) = 0$$

$$H_1 \left(\cancel{\theta} + \pi - \cancel{\theta} + \cancel{\theta} + \pi - \cancel{\theta}\right) = \frac{N_r i_r}{g}\left(\cancel{\theta} - \pi - \cancel{\theta}\right) + \frac{N_s i_s}{g}\left(\cancel{\theta} + \pi - \cancel{\theta}\right)$$

$$H_1 (2\pi) = \frac{N_r i_r}{g}(-\pi) + \frac{N_s i_s}{g}\pi$$

2010

$$H_1 = \frac{N_s i_s}{2g} - \frac{N_r i_r}{2g}$$

$$H_2 = H_1 + \frac{N_r i_r}{g} = \frac{N_s i_s}{2g} + \frac{N_r i_r}{2g}$$

$$H_3 = H_1 - \frac{N_s i_s}{g} + \frac{N_r i_r}{g} = -\frac{N_s i_s}{2g} + \frac{N_r i_r}{2g}$$

$$H_4 = H_1 - \frac{N_s i_s}{g} = -\frac{N_s i_s}{2g} - \frac{N_r i_r}{2g}$$

Faraday $\quad -v_s + 0 = -\frac{d}{dt}\lambda_s$

$$\lambda_s = N_s\left(\mu_0 H_1 \theta R l + \mu_0 H_2 (\pi - \theta) R l\right)$$

$$= \frac{\mu_0 N_s^2 \theta R l}{2g} i_s - \frac{\mu_0 N_s N_r \theta R l}{2g} i_r$$

$$+ \frac{\mu_0 N_s^2 (\pi - \theta) R l}{2g} i_s + \frac{\mu_0 N_s N_r (\pi - \theta) R l}{2g} i_r$$

$$= \frac{\mu_0 N_s^2 \pi R l}{2g} i_s + \frac{\mu_0 N_s N_r \pi R l}{2g}\left(1 - \frac{2\theta}{\pi}\right) i_r$$

$$= L_s i_s + L_m(\theta) i_r$$

$$\lambda_r = N_r \left(\mu_0 H_2 (\pi - \theta) R\ell + \mu_0 H_3 \theta R \ell \right)$$

$$= \frac{\mu_0 N_r N_s (\pi-\theta) R\ell}{2g} i_s + \frac{\mu_0 N_r^2 (\pi - \theta) R\ell}{2g} i_r$$

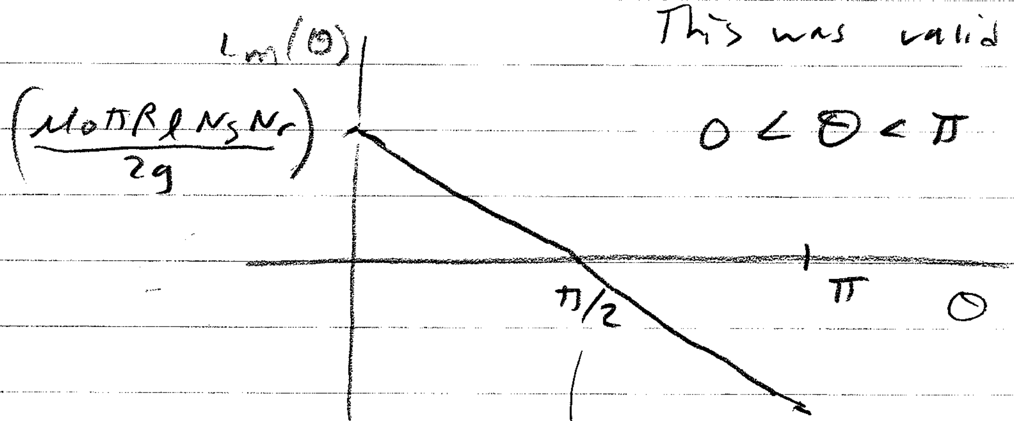
$$+ \frac{\mu_0 N_s N_r \theta R\ell}{2g} i_s + \frac{\mu_0 N_r^2 \theta R\ell}{2g} i_r$$

$$= \frac{\mu_0 N_s N_r \pi R\ell}{2g} \left(1 - \frac{2\theta}{\pi}\right) i_s + \frac{\mu_0 N_r^2 \pi R\ell}{2g} i_r$$

$$= L_m(\theta) i_s + L_r i_r$$

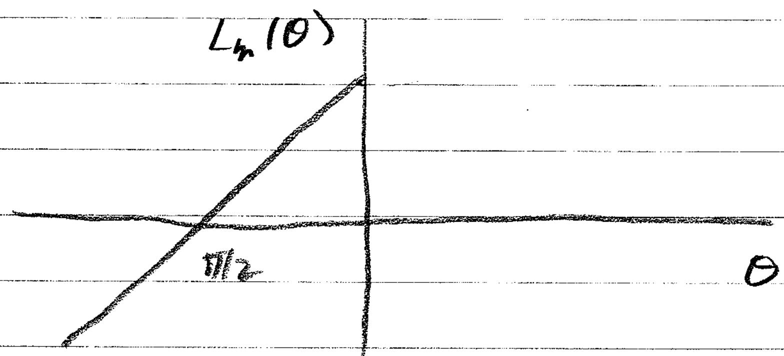
This was valid for

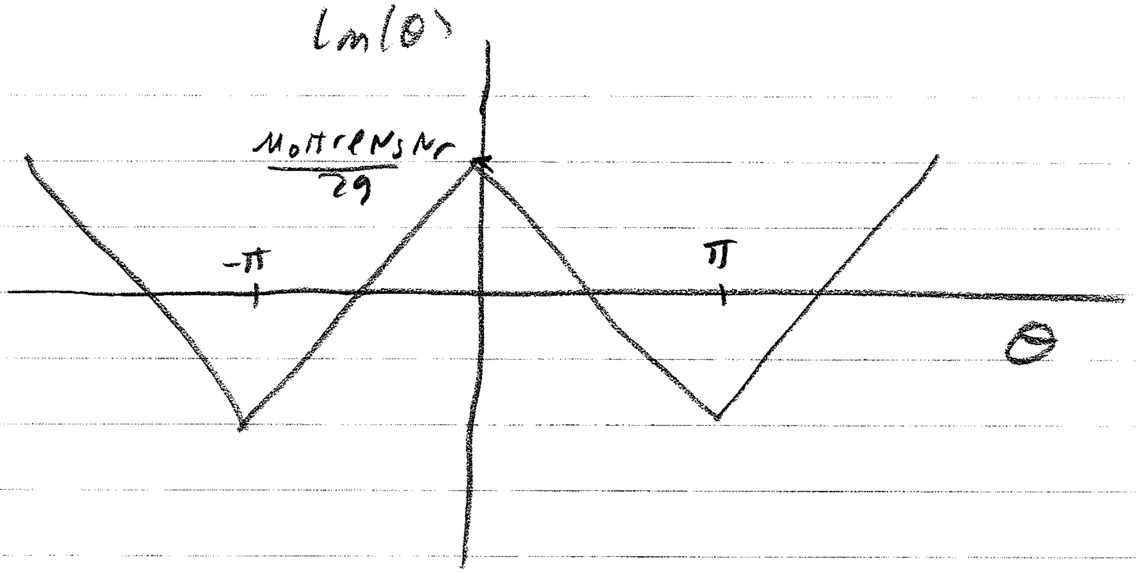
$0 < \theta < \pi$



(note error in book)

Can show by moving rotor and resolution





$$L_m(\theta) \approx M \cos\theta$$

STOP here

Fourier Series

$$
\begin{aligned}
f(t) \quad &= a_0 + a_1 \cos w_0 t + b_1 \sin w_0 t \\
\text{periodic} \quad & \quad + a_2 \cos 2w_0 t + b_2 \sin 2w_0 t \\
w_0 = 2\pi f_0 \quad & \quad + a_3 \cos 3w_0 t + b_3 \sin 3w_0 t \\
T = \frac{1}{f_0} \quad & \quad \vdots
\end{aligned}
$$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt \qquad \text{let } w_0 t = \theta$$

$$a_0 = \frac{1}{2\pi}\left[\int_0^{\pi}\left(\frac{\mu_0\pi r \ell N_s N_r}{2g}\right)\left(1-\frac{2\theta}{\pi}\right)d\theta + \int_{\pi}^{2\pi}\left(\frac{\mu_0\pi r \ell N_s N_r}{2g}\right)\left(\frac{2\theta}{\pi}-3\right)d\theta\right]$$

$$= \frac{1}{2\pi}\left[\frac{\mu_0\pi r \ell N_s N_r}{2g}\right]\left[\left[\theta - \frac{\theta^2}{\pi}\right]\Bigg|_0^{\pi} + \left[\frac{\theta^2}{\pi} - 3\theta\right]\Bigg|_{\pi}^{2\pi}\right] = 0$$

$$a_1 = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos\omega_0 t\,dt \qquad \text{let } \omega_0 t = \theta$$

$$a_1 = \frac{2}{2\pi}\,\frac{\mu_0\pi r l N_s N_r}{2g}\left[\int_0^{\pi}\left(1-\frac{2\theta}{\pi}\right)\cos\theta\,d\theta + \int_{\pi}^{2\pi}\left(\frac{2\theta}{\pi}-3\right)\cos\theta\,d\theta\right]$$

$$= \frac{\mu_0\pi r l N_s N_r}{2g}\left(\frac{1}{\pi}\right)\left[\left(\sin\theta - \frac{2}{\pi}(\cos\theta+\theta\sin\theta)\right)\Big|_0^{\pi} + \left(\frac{2}{\pi}(\cos\theta+\theta\sin\theta) - 3\sin\theta\right)\Big|_{\pi}^{2\pi}\right]$$

$$= \frac{\mu_0\pi r l N_s N_r}{2g}\left(\frac{1}{\pi}\right)\left[-\frac{2}{\pi}(-1-1) + \frac{2}{\pi}(1+1)\right]$$

$$= \frac{\mu_0\pi r l N_s N_r}{2g}\left(\frac{1}{\pi}\right)\left[\frac{8}{\pi}\right]$$

$$= \frac{\mu_0\pi r l N_s N_r}{2g}\left(\frac{8}{\pi^2}\right) \qquad \left(\text{Can show } b_1 = 0\right)$$

$$\boxed{L(\theta) \approx \frac{\mu_0\pi r l N_s N_r}{2g}\left(\frac{8}{\pi^2}\right)\cos\theta}$$

#2D

Mechanical Equations (Newton's 2nd Law)

Translation $\frac{dx}{dt} = v$ (force of electrical origin)

$$m \frac{dv}{dt} = \underbrace{f^e + f_{spring} + f_{damp.} + f_{ext}}_{\text{forces in } +x \text{ direction}}$$

$\left(\text{Power} = f v = f \frac{dx}{dt}\right)$

(Torque of electrical origin)

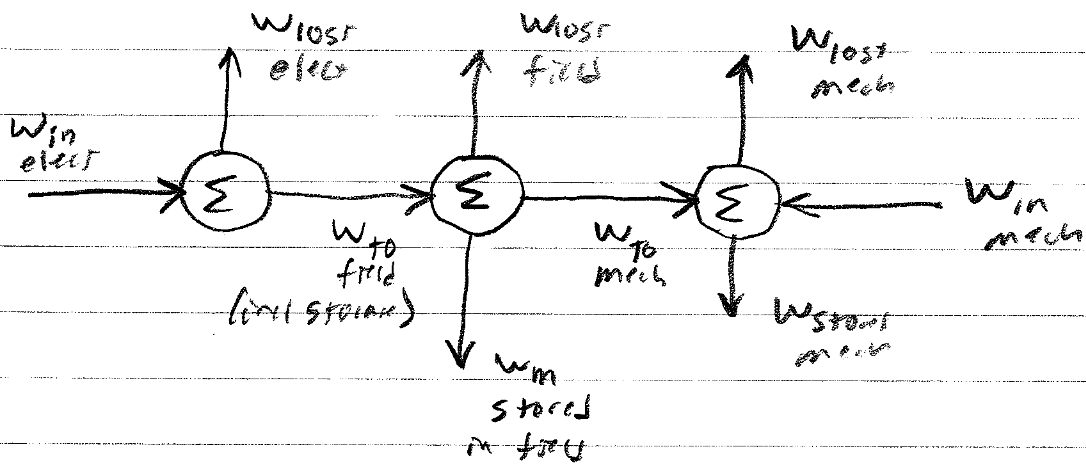
Rotational $\frac{d\theta}{dt} = \omega$

$$J \frac{d\omega}{dt} = \underbrace{T^e + T_{spring} + T_{damp} + T_{ext}}_{\text{torques in } +\theta \text{ direction}}$$

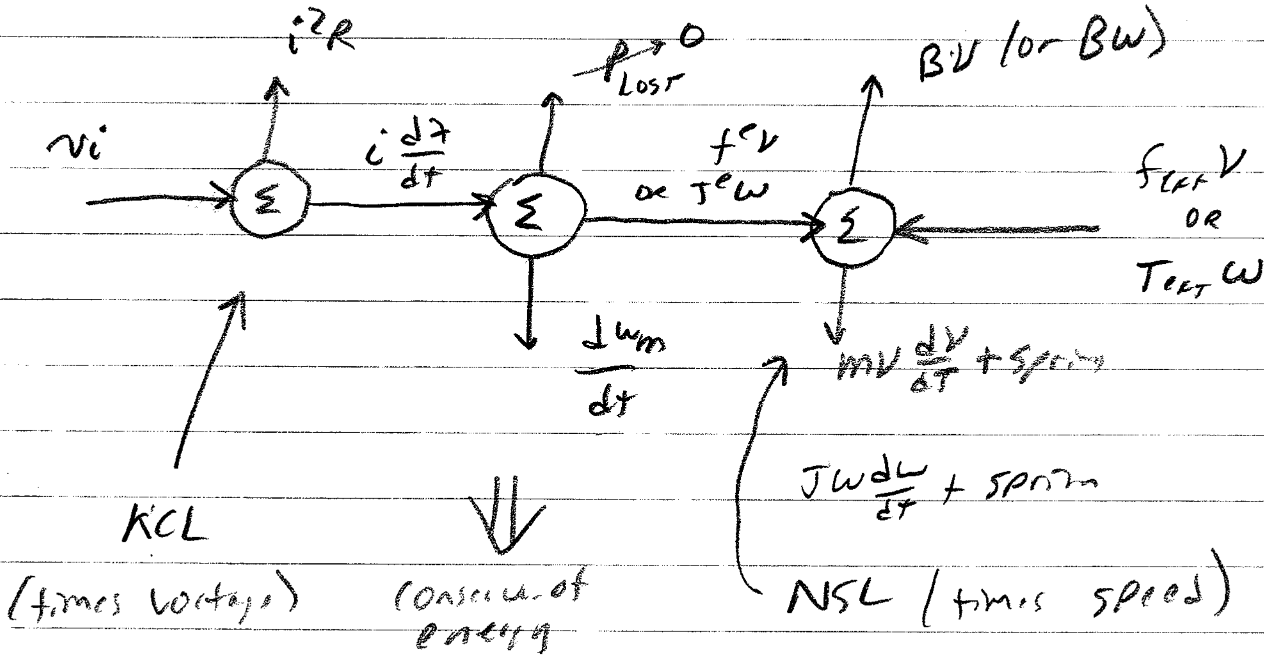
$\left(\text{Power} = T\omega = T \frac{d\theta}{dt}\right)$

Forces of Electric Origin

Start with Energy Conversion



differentiate $\frac{dW}{dt} = P$



$$\frac{dW_m}{dt} = i\frac{d\lambda}{dt} - f^e\frac{dx}{dt}$$

with choice of λ and x as independent variables,

$$\frac{dW_m(\lambda, x)}{dt} = \frac{\partial W_m(\lambda, x)}{\partial \lambda}\frac{d\lambda}{dt} + \frac{\partial W_m(\lambda, x)}{\partial x}\frac{dx}{dt}$$

So: $$i(\lambda, x) = \frac{\partial W_m(\lambda, x)}{\partial \lambda}$$

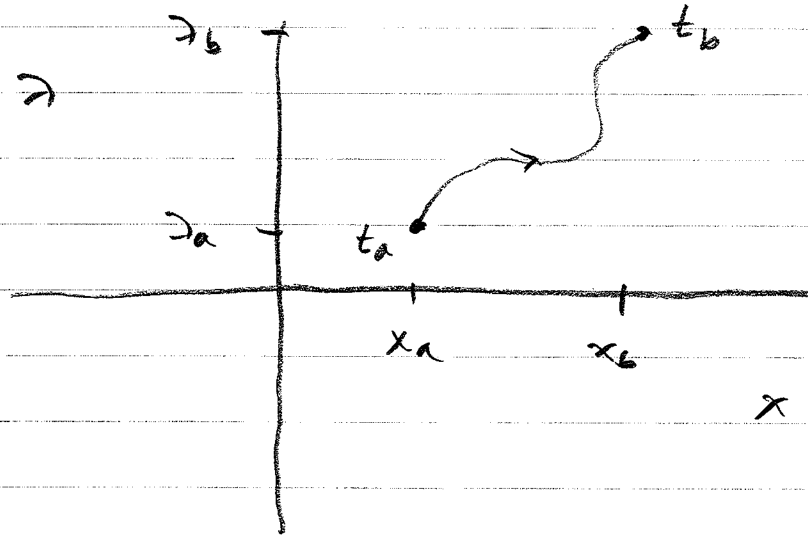
$$f^e(\lambda, x) = -\frac{\partial W_m(\lambda, x)}{\partial x}$$

Need to find $W_m(\lambda, x)$ to compute f^e

Similarly for rotation:

$$i(\lambda, \theta) = \frac{\partial W_m(\lambda, \theta)}{\partial \lambda}$$

$$T^e(\lambda, \theta) = -\frac{\partial W_m(\lambda, \theta)}{\partial \theta}$$



$$\int_{t_a}^{t_b} \frac{dw_m}{dt}\,dt = \int_{t_a}^{t_b} \left(i(\lambda, x)\frac{d\lambda}{dt} - f^e(\lambda, x)\frac{dx}{dt} \right) dt$$

Stop 2008

Change of variables

$t \to w_m$

$t \to \lambda$

$t \to x$

what is x doing during integral?

what is λ doing during integral?

$$\int_{W_{ma}}^{W_{mb}} dW_m = \int_{\lambda_a}^{\lambda_b} i(\lambda, x)\, d\lambda - \int_{x_a}^{x_b} f^e(\lambda, x)\, dx$$

For a conservative coupling field (no Hysteresis or eddy current losses) this integral is path independent

Choose a path:

1. Integrate x keeping λ constant at λ_a
2. Integrate λ keeping x constant at x_b

$$W_{mb} - \overset{0}{\cancel{W_{ma}}} = \int_{\lambda_a=0}^{\lambda_b} i(\lambda, x_b)\, d\lambda - \int_{x_a}^{x_b} f^e(\lambda_a, x)\, dx$$

Use $\lambda_a = 0$ (so $f^e = 0$) and assume $W_{ma} = 0$

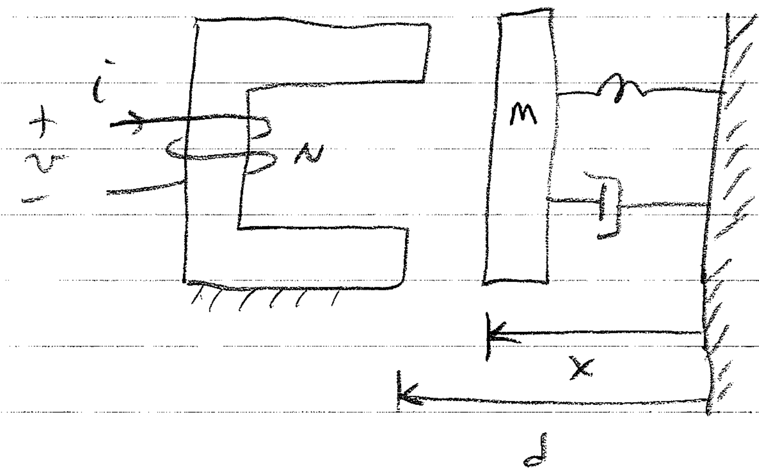
Let λ_b be any λ and x_b be any x

$$W_m(\lambda, x) = \int_0^{\lambda} i(\hat{\lambda}, x)\, d\hat{\lambda}$$

treated as $x = $ constant

Example

#21



$$\lambda = \frac{\mu_0 A N^2}{2(d-x)} i \qquad \text{OR} \qquad i = \frac{2(d-x)}{\mu_0 A N^2} \lambda$$

$$w_m(\lambda, x) = \int_0^{\lambda} \frac{2(d-x)\hat{\lambda}}{\mu_0 A N^2} d\hat{\lambda}$$

x treated as constant

$$= \frac{(d-x)\lambda^2}{\mu_0 A N^2}$$

$$f^e(\lambda, x) = -\frac{\partial w_m(\lambda, x)}{\partial x} = +\frac{\lambda^2}{\mu_0 A N^2}$$

Recall other condition: $i(\lambda, x) = \dfrac{\partial w_m(\lambda, x)}{\partial \lambda} = \dfrac{2(d-x)\lambda}{\mu_0 A N^2}$ ✓

When we have found $f^e(\lambda, x)$, we can also substitute for λ to get:

$$f^e(i,x) = +\frac{1}{\mu_0 A N^2}\left(\frac{\mu_0 A N^2 i}{2(d-x)}\right)^2$$

$$= +\frac{\mu_0 A N^2 i^2}{4(d-x)^2}$$

There is a direct way to get this same result.

<u>Define coenergy</u>

$$W_m' \triangleq \lambda i - W_m$$

Using i and x as independent variables,

$$\frac{dW_m'(i,x)}{dt} = \frac{\partial W_m'(i,x)}{\partial i}\frac{di}{dt} + \frac{\partial W_m'(i,x)}{\partial x}\frac{dx}{dt}$$

$$\frac{dW_m'(i,x)}{dt} = \lambda\frac{di}{dt} + i\frac{d\lambda}{dt} - \frac{dW_m}{dt}$$

$$= \lambda\frac{di}{dt} + \cancel{i\frac{d\lambda}{dt}} - \left(\cancel{i\frac{d\lambda}{dt}} - f^e\frac{dx}{dt}\right)$$

Equate coefficients,

$$\lambda(i,x) = \frac{\partial w_m'(i,x)}{\partial i}$$

$$f^e(i,x) = \frac{\partial w_m'(i,x)}{\partial x}$$

How do we find $w_m'(i,x)$?

Method #1

$$w_m'(i,x) = \lambda(i,x)\, i - w_m(\lambda(i,x), x)$$

Method #2

$$\frac{dw_m'}{dt} = \lambda \frac{di}{dt} + f^e \frac{dx}{dt}$$ (Choose i & x as independent variables)

$$\int_{t_a}^{t_b} \frac{dw_m'}{dt}\, dt = \int_{t_a}^{t_b} \left(\lambda \frac{di}{dt} + f^e \frac{dx}{dt} \right) dt$$

Change of variables (using i & x as independent)

$$w'_{m_b} - w'_{m_a} = \int_{i_a}^{i_b} \lambda(i,x)\,di + \int_{x_a}^{x_b} f^e(i,x)\,dx$$

Choose a path

1. Integrate x keeping i constant at i_a

2. Integrate λ keeping x constant at x_b

$$w'_{m_b} - w'_{m_a} = \int_{i_a}^{i_b} \lambda(i, x_b)\,di + \int_{x_a}^{x_b} f^e(i_a, x)\,dx$$

use $i_a = 0$ (so $f^e = 0$) and assume $w'_{m_a} = 0$

Let i_b be any i and x_b be any x

$$w'_m(i,x) = \int_0^{i} \lambda(\hat{i}, x)\Big|_{x=\text{constant}}\,d\hat{i}$$

Example

$$\lambda = \frac{\mu_0 A N^2}{2(d-x)} i$$

$$w_m'(i,x) = \int_0^i \frac{\mu_0 A N^2 \hat{i}}{2(d-x)} \Bigg|_{x=\text{constant}} d\hat{i}$$

$$= \frac{\mu_0 A N^2 i^2}{4(d-x)}$$

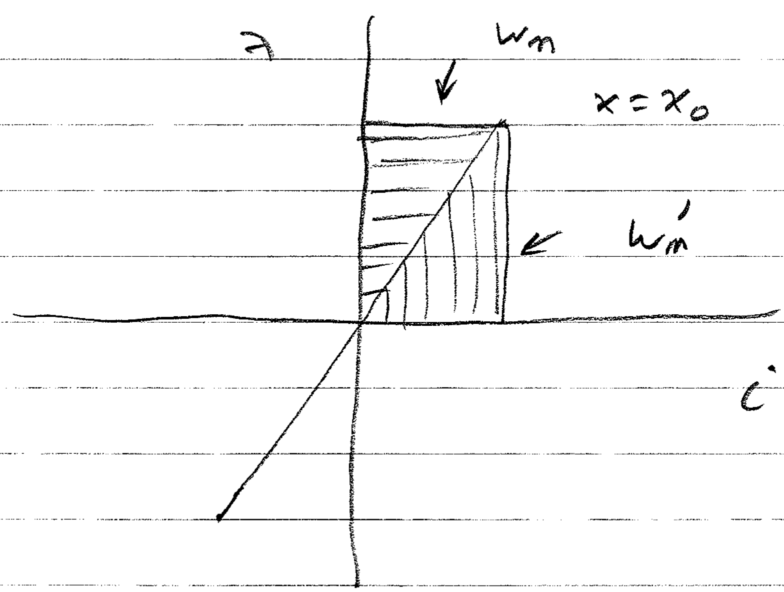
$$f^e(i,x) = + \frac{\partial w_m'(i,x)}{\partial x} = + \frac{\mu_0 A N^2 i^2}{4(d-x)^2}$$

Energy vs Coenergy

$$w_m(\lambda, x) = \int_0^{\lambda} i(\hat{\lambda}, x)\Big|_{x = \text{constant}} d\hat{\lambda}$$

$$w_m'(i, x) = \int_0^{i} \lambda(\hat{i}, x)\Big|_{x = \text{constant}} d\hat{i}$$

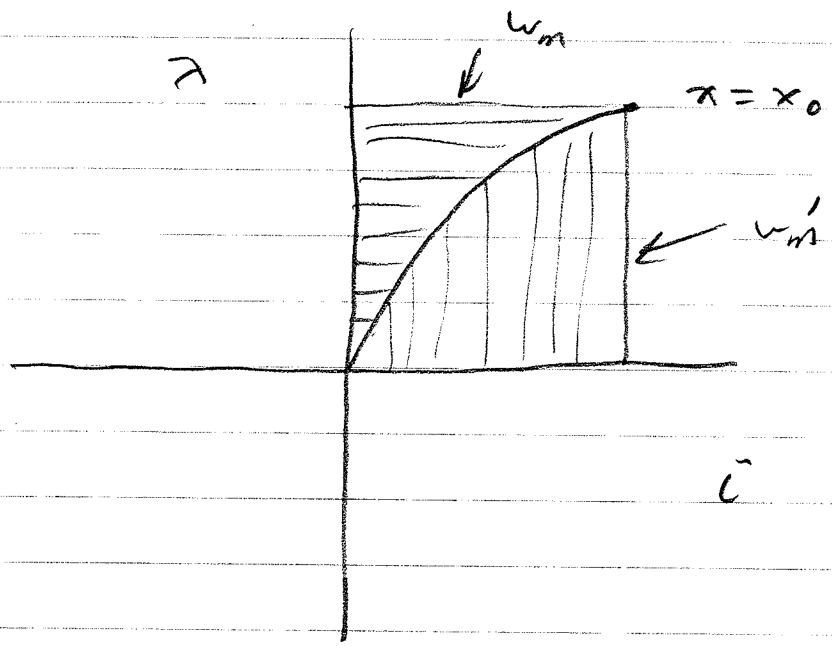
Suppose $\lambda = L(x)\, i$ (linear)



$W_m = W_m'$ for linear systems

and note $\lambda i = W_m + W_m'$ Checks ✓

$\lambda = f(i, x)$ (non linear)



$W_m \neq W_m'$ for nonlinear

but still $\lambda i = W_m + W_m'$ ✓

Multi-port systems (#22)

$$\lambda_1 = L_{11}(x)i_1 + L_{12}(x)i_2$$

$$\lambda_2 = L_{21}(x)i_1 + L_{22}(x)i_2$$

Path from $0 \to i_1$ $0 - i_2$ $x_0 \to x$

must be path independent ($L_{12} = L_{21}$ for linear)

Choose path x first while i_1 & i_2 are zero

i_1 next while $x = \text{const}$ $i_2 = 0$

i_2 last while $x = \text{const}$ $i_1 = i_1$

$$w_m'(i_1, i_2, x) = \int_0^{i_1} \lambda_1 \Big|_{\substack{i_2 = 0 \\ x = \text{const}}} di_1 + \int_0^{i_2} \lambda_2 \Big|_{\substack{i_1 = i_1 \ \text{const} \\ x = \text{const}}} di_2 + \int_{x_0}^{x} \cancel{f^e} \Big|_{\substack{i_1 = 0 \\ i_2 = 0}} dx$$

$$= \frac{1}{2} L_{11}(x) i_1^2 + L_{21}(x) i_1 i_2 + \frac{1}{2} L_{22}(x) i_2^2$$

$$f^e(i_1, i_2, x) = \frac{\partial w_m'(i_1, i_2, x)}{\partial x}$$

Example (Rotational)

$$\lambda_s = L_s i_s + m \cos\theta \, i_r$$

$$\lambda_r = m \cos\theta \, i_s + L_r i_r$$

$$W_m'(i_s, i_r, \theta) = \int_0^{i_s} L_s \hat{i}_s \, d\hat{i}_s + \int_0^{i_r} \left(m \cos\theta \, i_s + L_r \hat{i}_r \right) d\hat{i}_r$$

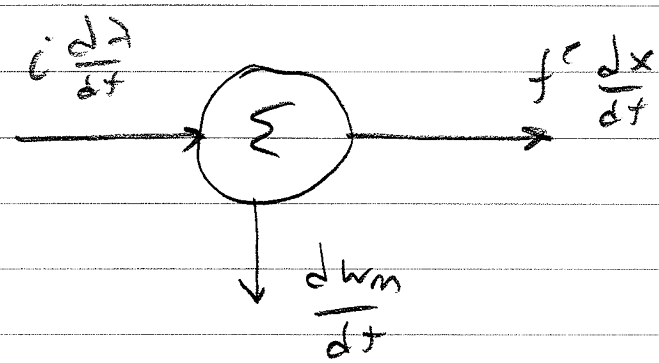
$$= \frac{1}{2} L_s i_s^2 + m \cos\theta \, i_s i_r + \frac{1}{2} L_r i_r^2$$

$$T^e(i_s, i_r, \theta) = \frac{\partial W_m'(i_s, i_r, \theta)}{\partial \theta}$$

$$= -m \sin\theta \, i_s i_r$$

Book Sec. 4.7 STOP 2006

Energy Conversion Cycles



$$\text{Energy From Electrical (EFE)} = \int_{t_a}^{t_b} i\frac{d\lambda}{dt}\Big|_{\text{along path}} dt$$

$$\underset{a\to b}{EFE} = \int_{\lambda_a}^{\lambda_b} i\Big|_{\text{along path}} d\lambda$$

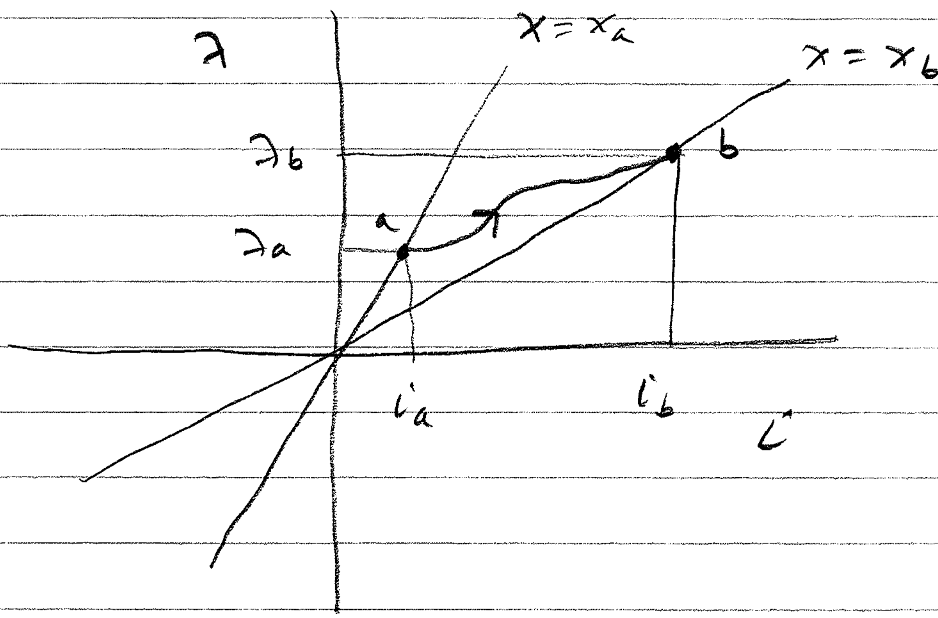
$$\text{Energy From Mechanical (EFM)} = -\int_{t_a}^{t_b} f^e\frac{dx}{dt}\Big|_{\text{along path}} dt$$

$$\underset{a\to b}{EFM} = -\int_{x_a}^{x_b} f^e\Big|_{\text{along path}} dx$$

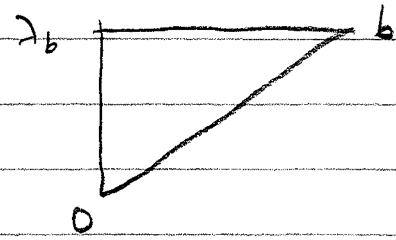
$$w_{ma} = \int_0^{\lambda_a} i\Big|_{x=x_a} d\lambda \qquad w_{mb} = \int_0^{\lambda_b} i\Big|_{x=x_b} d\lambda$$

$$w_{m_b} - w_{m_a} = \underset{a\to b}{EFE} + \underset{a\to b}{EFM}$$

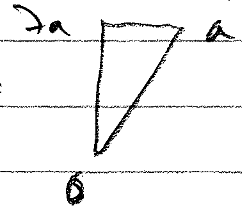
Example



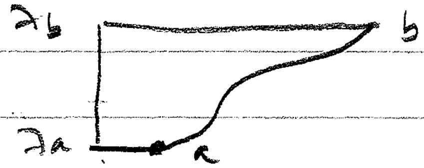
$w_{m_b} =$



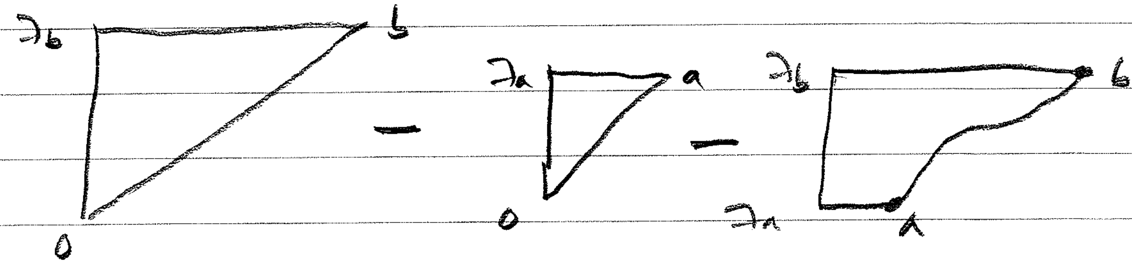
$w_{m_a} =$



$\underset{a\to b}{EFE} =$



Effm
$a \to b$ =

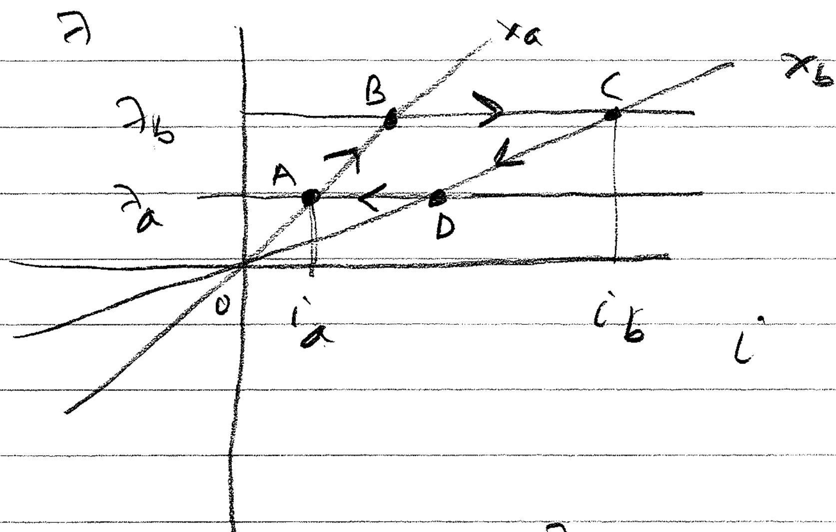


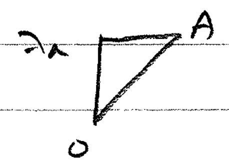
#23 cancel EOH

Stop 2006

#24

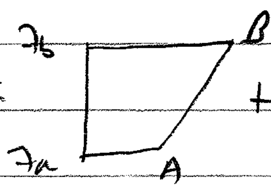
Complete cycle

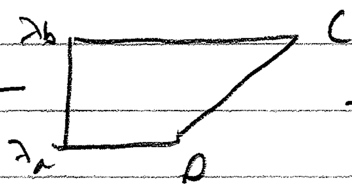


W_m begin cycle $= \displaystyle\int_0^{\lambda_a} i\Big|_{x=x_a} d\lambda =$ 

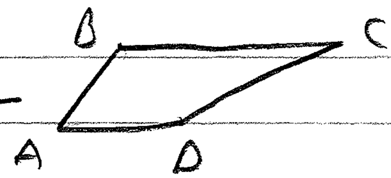
W_m end cycle = Same

So: $\underset{cycle}{EFE} + \underset{cycle}{EFM} = 0$

$\underset{cycle}{EFE} =$ 

$+ 0 -$ 

$+ 0$

$= -$ 

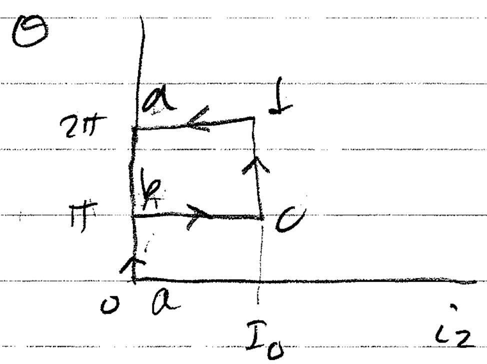
(This is a generator)

Example 4.14 Book

$$\lambda_1 = L_{11} i_1 + m\cos\theta\, i_2$$

$$\lambda_2 = m\cos\theta\, i_1 + L_{22} i_2$$

Path



$i_1 = I_0$

Find: EFM_{cycle}, EFE_{cycle}

Soln:

$$W_m' = \frac{1}{2} L_{11} i_1^2 + m\cos\theta\, i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

$$T^e = -m\sin\theta\, i_1 i_2$$

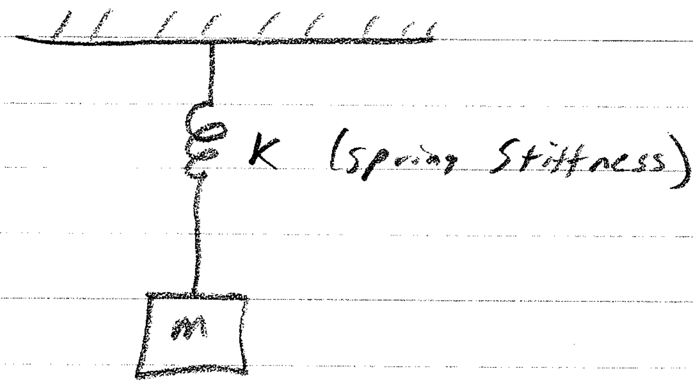
$$EFM_{cycle} = -\int_0^{2\pi} T^e d\theta = -\int_0^{\pi} 0\, d\theta - \int_{\pi}^{2\pi} (-m\sin\theta\, I_0^2)\, d\theta$$

$$= -m\cos\theta\, I_0^2 \Big|_{\pi}^{2\pi} = -mI_0^2 - (mI_0^2) = -2mI_0^2$$

$EFM_{cycle} < 0$ so motor

Stop 2006

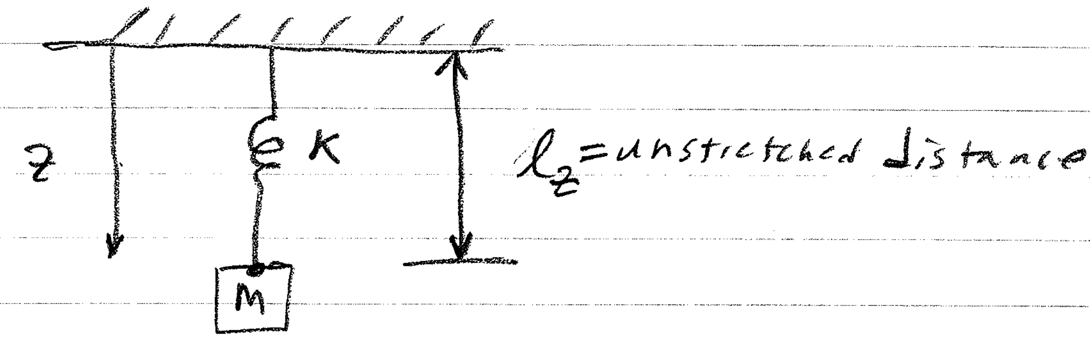
Dynamics of lumped mechanical systems



Select a distance measure for reference

Choices: Down from ceiling
Down from "unstretched" position
Down from equilibrium position
To top of mass
To bottom of mass
To center of gravity of mass

Look at Spring with no mass



z is measured to point of connection (top of mass)

Add mass to end of spring

$$\left.\begin{aligned} \frac{dz}{dt} &= v \\ m\frac{dv}{dt} &= mG - K(z - l_z) \end{aligned}\right\} \quad z = l_z + \frac{mG}{K} \text{ is equilibrium}$$

Could define $y \triangleq z - l_z$

$$\left.\begin{aligned} \frac{dy}{dt} &= v \\ m\frac{dv}{dt} &= mG - Ky \end{aligned}\right\} \quad y = \frac{mG}{K} \text{ is equilibrium}$$

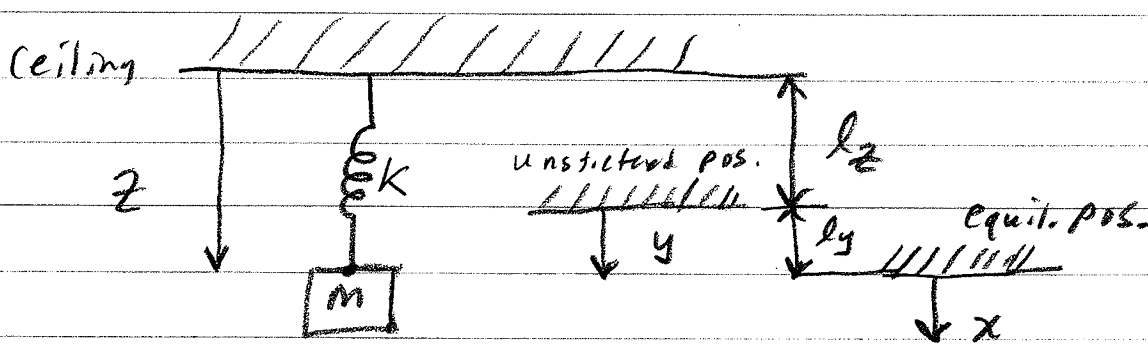
Now define $l_y \triangleq \frac{mG}{K}$

then

$$\left.\begin{aligned} \frac{dy}{dt} &= v \\ m\frac{dv}{dt} &= -k(y-l_y) \end{aligned}\right\} \quad y = l_y \text{ is equilibrium}$$

Could also define $x \triangleq y - l_y$

$$\left.\begin{aligned} \frac{dx}{dt} &= v \\ m\frac{dv}{dt} &= -kx \end{aligned}\right\} \quad x = 0 \text{ is equilibrium}$$

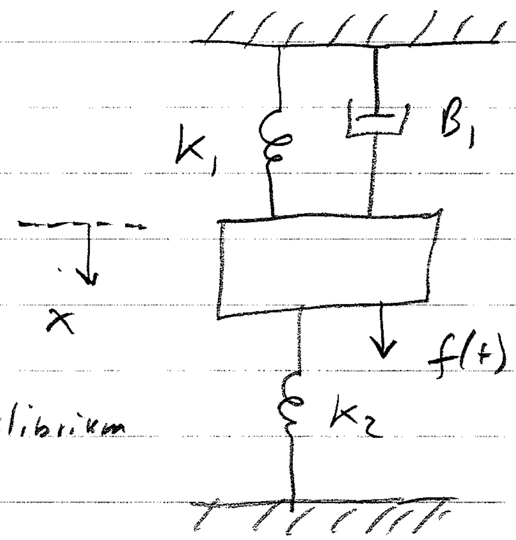


In this figure, $z = z^e = l_z + l_y$

$$y = y^e = l_y$$

$$x = x^e = 0$$

Example



$x=0$ is equilibrium

$$\frac{dx}{dt} = v$$

$$m\frac{dv}{dt} = -K_1 x - K_2 x - Bv + f(t)$$

Note: if $f(t)=0$ $x=0$ is equilibrium

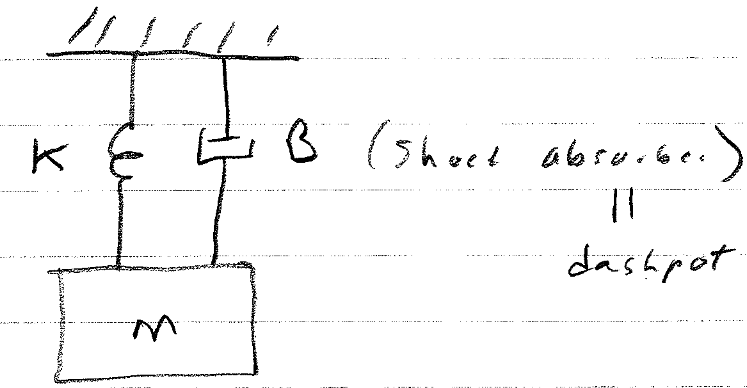
if $x<0$ K_1 is compressed, so pushes down ✓
K_2 is stretched, so pulls down ✓

if $x>0$ K_1 is stretched, so pulls up ✓
K_2 is compressed, so pushes up ✓

if $v<0$ B_1 pushes down ✓

if $v>0$ B_1 pulls up ✓

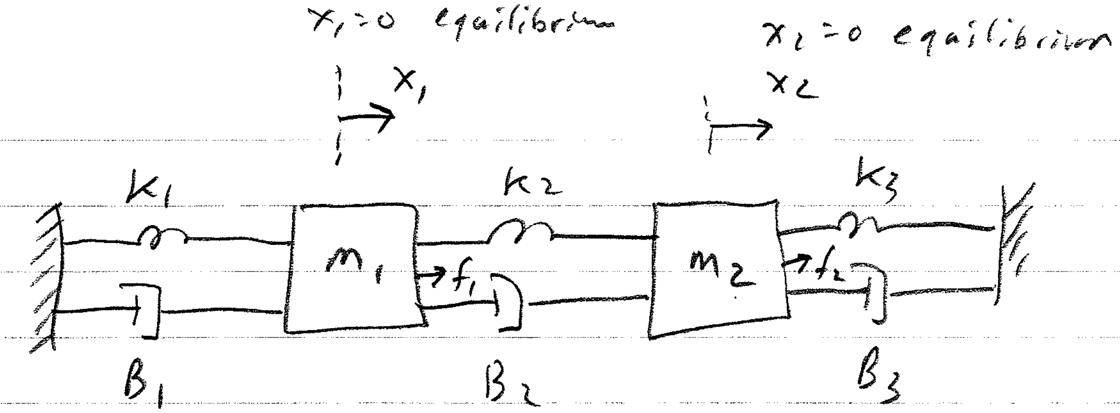
#25



measure x down from equilibrium position

$$\frac{dx}{dt} = v$$

$$m\frac{dv}{dt} = -kx - Bv$$



$$\frac{dx_1}{dt} = v_1$$

$$m_1 \frac{dv_1}{dt} = -K_1 x_1 - K_2(x_1 - x_2) - B_1 v_1 - B_2(v_1 - v_2) + f_1(t)$$

$$\frac{dx_2}{dt} = v_2$$

$$m_2 \frac{dv_2}{dt} = -K_3 x_2 - K_2(x_2 - x_1) - B_3 v_2 - B_2(v_2 - v_1) + f_2(t)$$

Let f_1 = unit step F_1 , f_2 = unit step F_2

Solve differential equations for $x_1\ v_1\ x_2\ v_2$
as function of time → use math 285 (easy – linear)

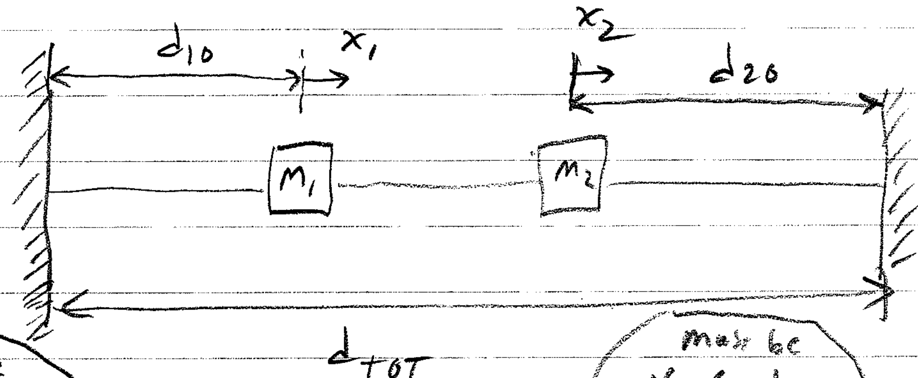
As $t \to \infty$ will be stable "dc" equilibrium wit $v_1 = v_2 = 0$

$$0 = -k_1 x_{1_{ss}} - k_2(x_{1_{ss}} - x_{2_{ss}}) + F_1$$
$$0 = -k_3 x_{2_{ss}} - k_2(x_{2_{ss}} - x_{1_{ss}}) + F_2$$

$$\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \begin{bmatrix} x_{1_{ss}} \\ x_{2_{ss}} \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

Note: There are physical limits on the possible values of x_1 and x_2

i.e.



must be $x_1 \geq -d_{10}$

must be $x_2 \leq d_{20}$

$d_{TOT} - d_{10} - d_{20}$ = gap between masses (must be positive)

so; since $x_1 > 0$ makes gap smaller and since $x_2 > 0$ makes gap bigger

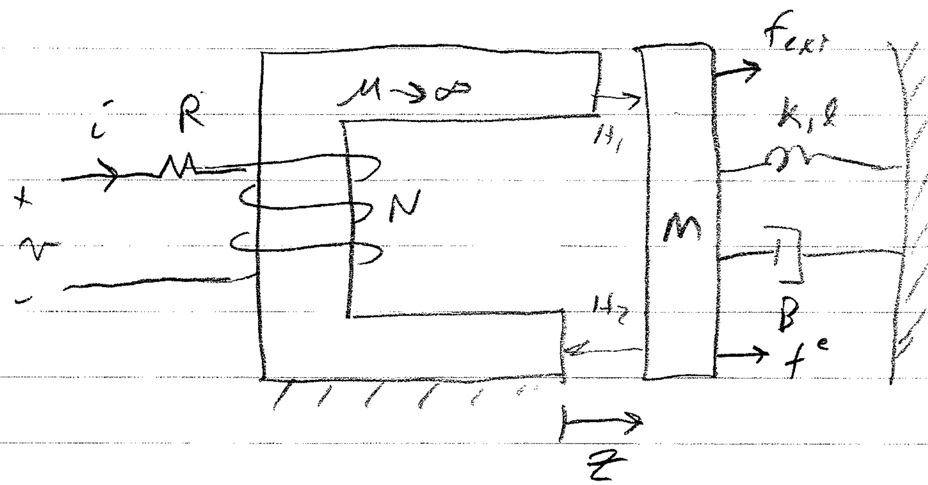
must be
$$x_1 - x_2 \leq d_{TOT} - d_{10} - d_{20}$$

These constraints must be satisfied during transient also

Actually, considering the lengths of masses, springs, dashpots, these limits would be even more constrained.

#26

State-Space models



ℓ = zero force distance for z

$$H_1 z - H_2 z = Ni \qquad -\mu_0 H_1 A + \mu_0 H_2 A = 0$$

$$H_1 = H_2 = \frac{Ni}{2z} \qquad B_1 = \frac{\mu_0 N i}{2z} \qquad \phi = \frac{\mu_0 A N i}{2z}$$

$$\lambda = \frac{\mu_0 A N^2}{2z} i \qquad w_m' = \frac{\mu_0 A N^2}{4z} i^2 \qquad f^e = -\frac{\mu_0 A N^2}{4z^2} i^2$$

KVL $\quad v = iR + \frac{d\lambda}{dt} = iR + \frac{\mu_0 A N^2}{2z}\frac{di}{dt} - \frac{\mu_0 A N^2}{2z^2}\frac{dz}{dt} i$

NSL $\quad \frac{dz}{dt} = v$

$$m\frac{dv}{dt} = f_{ext} + f^e - K(z - \ell) - Bv$$

Now put in state-space form

$$\frac{di}{dt} = \frac{2z}{\mu_0 A n^2}\left(v - iR + i\frac{\mu_0 A n^2 \nu}{2z^2}\right)$$

$$\frac{dz}{dt} = \nu \qquad \boxed{\text{Require } z > 0}$$

$$\frac{d\nu}{dt} = \frac{1}{m}\left(f_{ext} - \frac{\mu_0 A n^2 i^2}{4z^2} - K(z-\ell) - \beta\nu\right)$$

Let $x_1 = i$ $\qquad \dot{x}_1 = f_1(x_1, x_2, x_3, u_1, u_2)$

$x_2 = z$

$x_3 = \nu$ $\qquad \dot{x}_2 = f_2(x_1, x_2, x_3, u_1, u_2)$

$u_1 = v$

$u_2 = f_{ext}$ $\qquad \dot{x}_3 = f_3(x_1, x_2, x_3, u_1, u_2)$

OR $\qquad \dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u})$

Each dynamic state and input have initial values

$x_1(0) = x_1^0 \qquad u_1(0) = u_1^0$

$x_2(0) = x_2^0 \qquad u_2(0) = u_2^0$

$x_3(0) = x_3^0$

Equilibrium

$\dot{\underline{x}} = 0$ so $\underline{f} = 0$

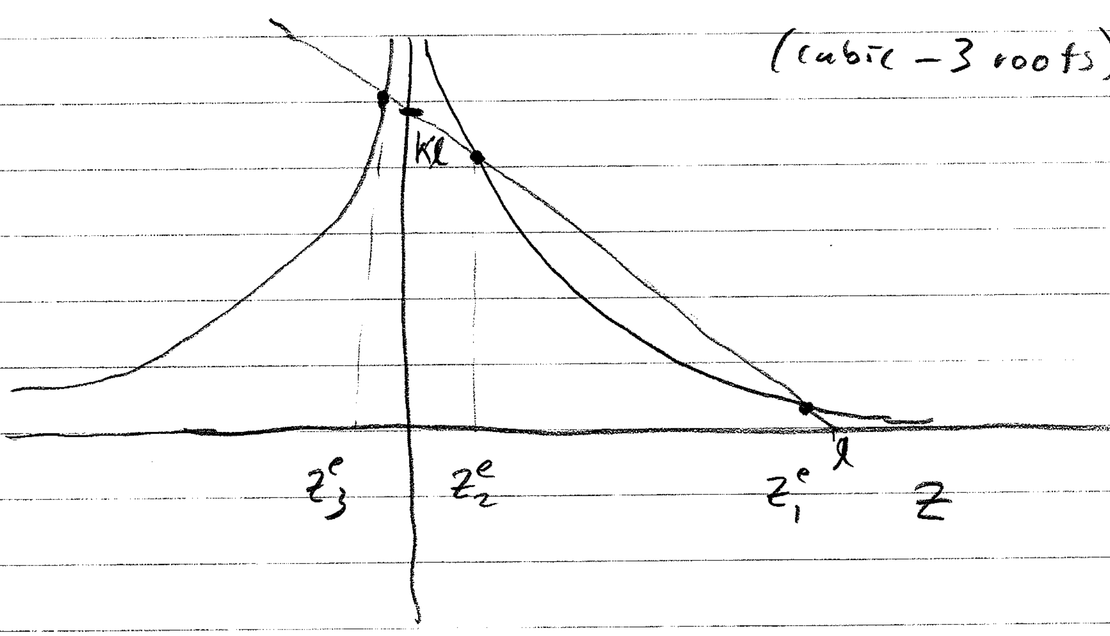
$0 = v - i^e R \Rightarrow i^e = \frac{v}{R}$

$0 = \nu^e$

$$0 = f_{ext} - \frac{\mu_0 A N^2 i^{e2}}{4 z^{e2}} - k(z^e - \ell)$$

let $f_{ext} = 0$ $\quad 0 = -\mu_0 A N^2 i^{e2} - 4 z^{e3} k + 4 z^{e2} k\ell$

(cubic – 3 roots)



z_1^e stable because if z moves a little bigger, f^e will dominate & bring back

z_2^e unstable because if z moves a little bigger, spring will dominate & make z bigger (move to z_1^e)

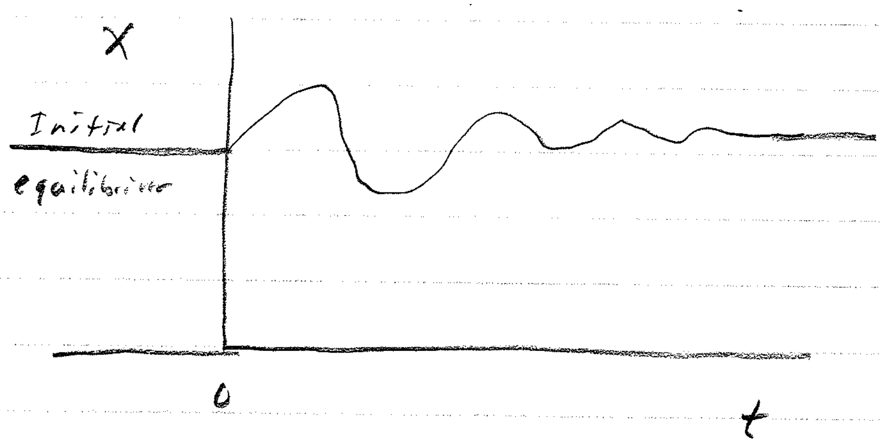
z_3^e impossible

Dynamic Responses

1. Start at an equilibrium point (may be Stable or may be Unstable)

2. Apply a disturbance

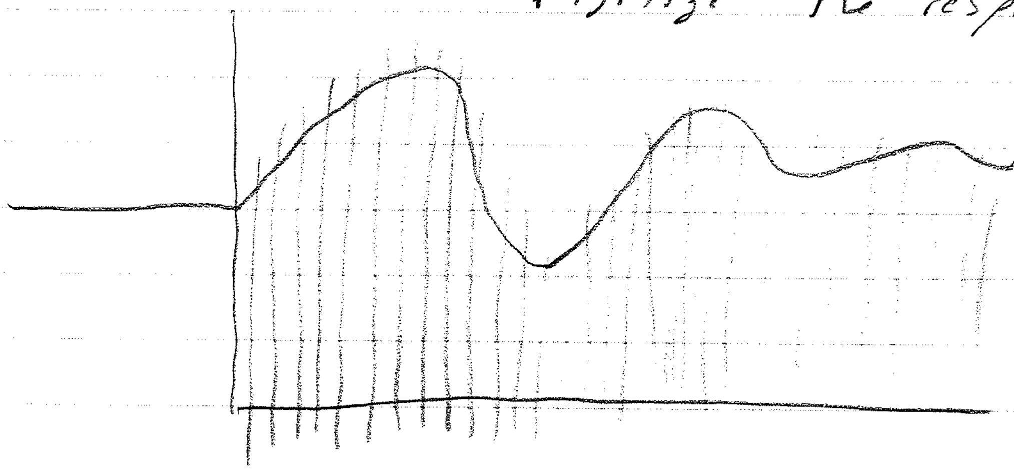
3. See what happens



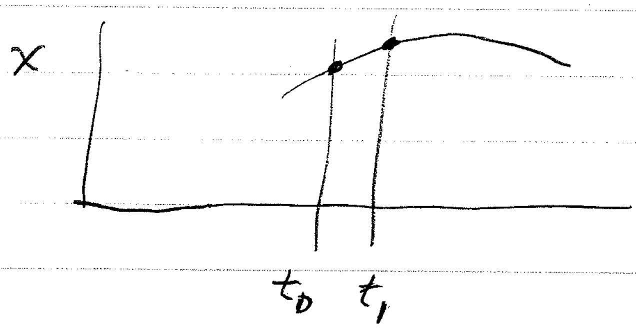
How can we compute (predict) this?

Divide time into very small segments (Steps)

"Digitize" the response



Use small enough time step so that response between end points is almost a straight line



Linear prediction

$$x(t_1) \approx x(t_0) + \left.\frac{dx}{dt}\right|_{t_0} (t_1 - t_0)$$

OR Think of this as a taylor series in time about t_0

$$x(t) = x(t_0) + \left.\frac{dx}{dt}\right|_{t_0} (t - t_0) + \underbrace{\frac{1}{2} \left.\frac{d^2x}{dt^2}\right|_{t_0} (t - t_0)^2 + \cdots}_{\text{higher order terms}}$$

neglect h.o.t.

This is Euler's explicit method

Example

Given: $$\frac{dx}{dt} = -5x^2 + u$$

$u=0$ & $x=0$ until time $t=0$ seconds

then $u = 10$ at $t=0$

Find response

x cannot change instantaneously
u can " "

Solve:

$$\frac{dx}{dt} = -5x^2 + 10 \qquad x(0) = 0$$

Note: for $t<0$, $x=0$ (initial equilibrium)

Note: new equilibrium is $x = \pm 1.414$

which one will it go to?

use Euler's method with time step of .01 sec

$x(0)=0$

$$x(.01) = x(0) + \left.\frac{dx}{dt}\right|_{t=0} .01 = 0 + (-5 \times 0^2 + 10) .01 = 0.1$$

$$x(.02) = x(.01) + \left.\frac{dx}{dt}\right|_{t=.01} .01 = 0.1 + (-5 \times (0.1)^2 + 10) .01 = 0.1995$$

$$x(.03) = x(.02) + \left.\frac{dx}{dt}\right|_{t=.02} .01 = 0.1995 + (-5 \times (.1995)^2 + 10) .01$$

going towards $x = 1.414$