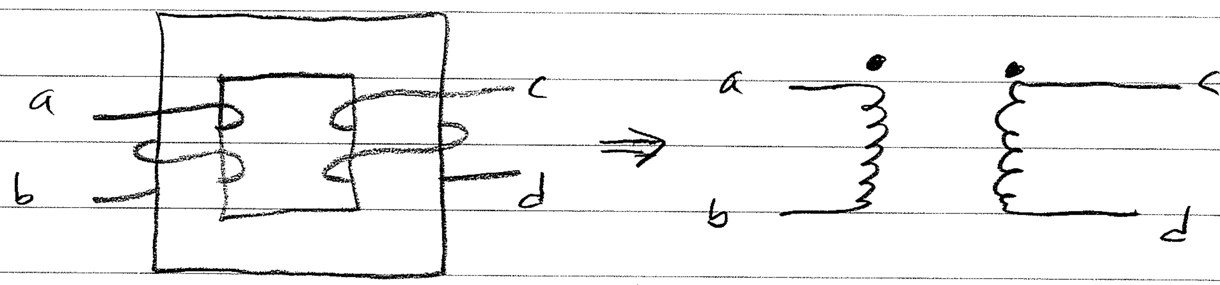
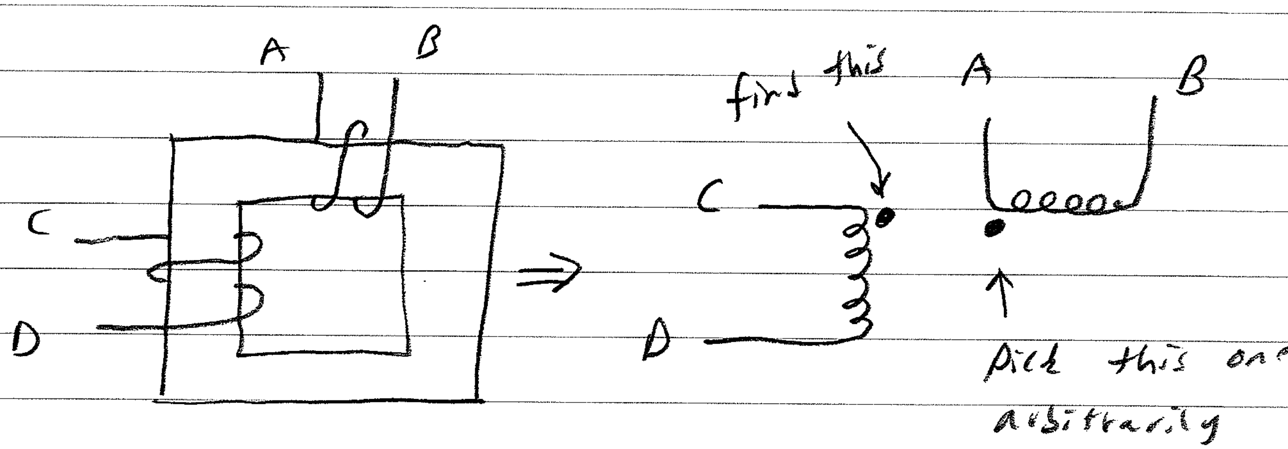
Polarity marks (The dot convention)

The dots are only needed when there are two or more coils.

The dots tell you how the coil is "wound"

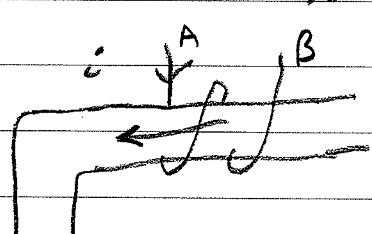




TO find the second dot:

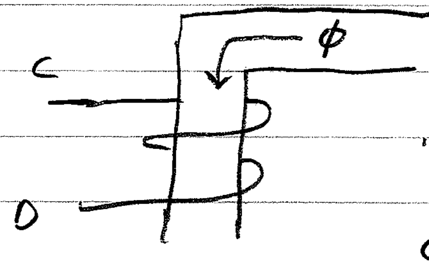
1. Inject current into "first dot" and use Right Hand Rule to find flux direction in iron

i.e.



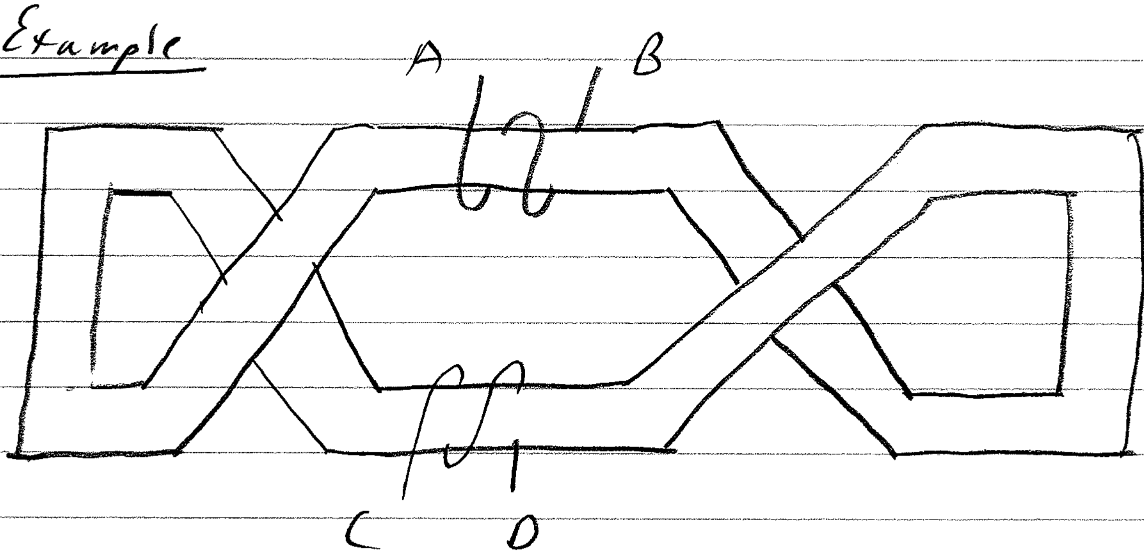
2. Put "Second dot" on the terminal of the other coil whose injection current gives the same flux direction of iron

i.e.



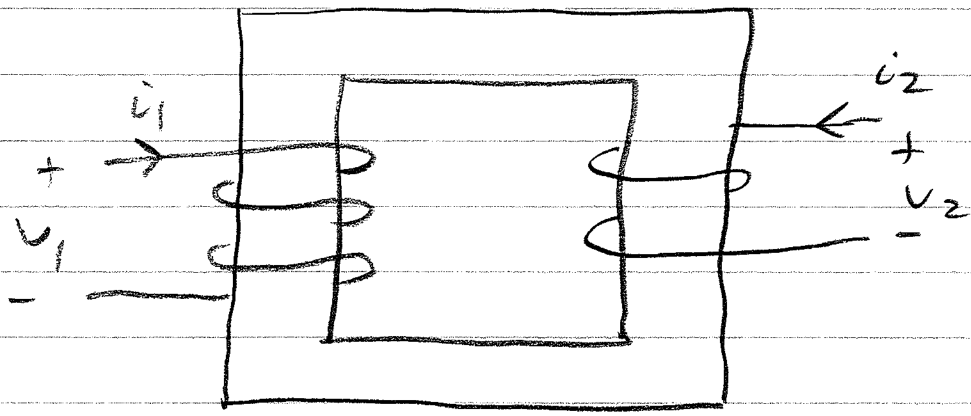
"Second dot" goes on C because if you inject current into C, it will create ϕ in same direction

Example



1. Pick terminal A for first dot
2. Inject current into A flux goes $\longrightarrow$ through iron of top coil and $\longleftarrow$ through bottom coil.
3. To get flux due to bottom coil to go $\longleftarrow$, inject current into C : so dot on C

Writing equations



With resistance included,

$$v_1 = i_1 R_1 + \frac{d\lambda_1}{dt} \qquad v_2 = i_2 R_2 + \frac{d\lambda_2}{dt}$$

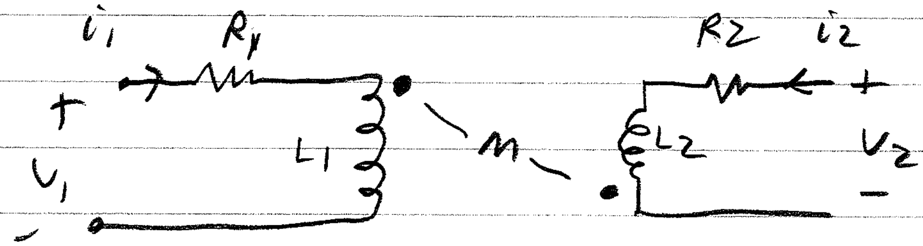
magnetic analysis would show that

$$\left.\begin{aligned} \lambda_1 &= L_1 i_1 - m i_2 \\ \lambda_2 &= -m i_1 + L_2 i_2 \end{aligned}\right\}$$ where L_1 L_2 m are all positive

So: $$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} - m \frac{di_2}{dt}$$

$$v_2 = i_2 R_2 - m \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

The "circuit model" for this is:



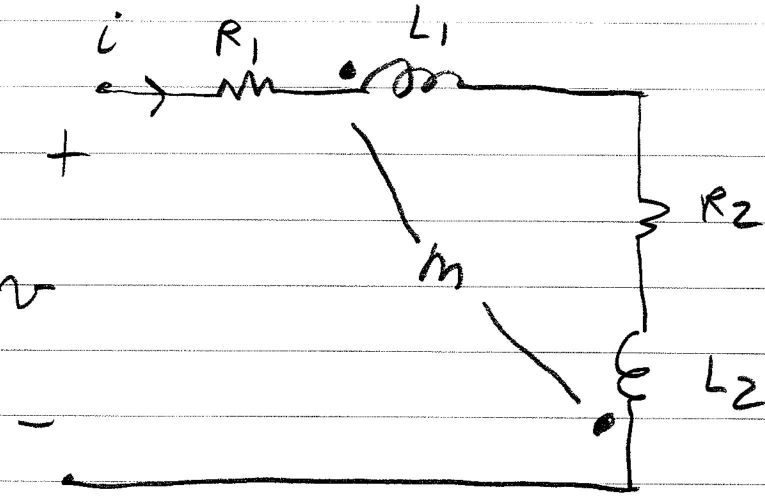
$$v_1 = \underbrace{i_1 R_1 + L_1 \frac{di_1}{dt}}_{\text{nothing to do with dots}} - M \frac{di_2}{dt}$$

minus because we are summing drops in left KVL and we want drop from "dot" to "undot"

So:

The mutual term will be M times the derivative of the current entering the dot of coil 2. This is $-i_2$

Example



(Always assume L_1 L_2 M are positive)

$$-v + \underbrace{R_1 i + L_1 \frac{di}{dt}}_{\text{do not need dots for this}} - M\frac{di}{dt} + \underbrace{R_2 i + L_2 \frac{di_2}{dt}}_{\text{do not use dots for this}} - M\frac{di}{dt}$$

minus because we want voltage drop from dot to undot of coil 1, so we use the current "into" the dot of coil 2 - which is $-i$

minus because we want voltage drop from undot to dot of coil 2, so use the current "into" the undot of coil 1 - which is $-i$

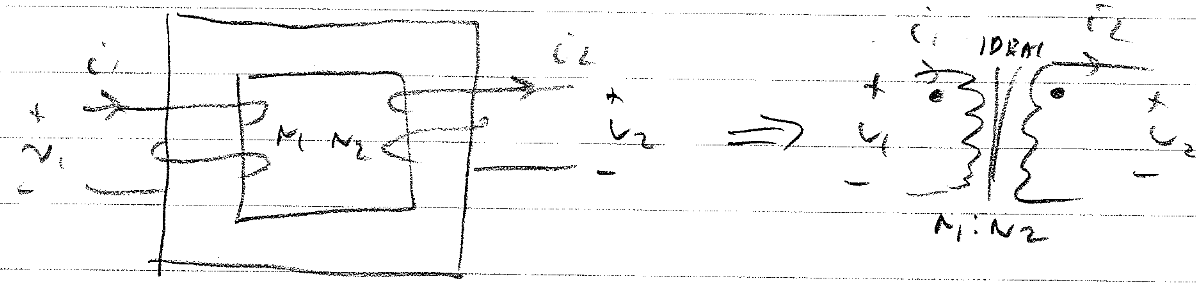
Go through Example 3.6 if time allows

IDEAL TRANSFORMER

Derive ideal model

$\sigma = \infty$

$\mu = \infty$



$$\boxed{N_1 i_1 = N_2 i_2} \qquad \boxed{v_2 = \frac{N_2}{N_1} v_1}$$

$$a = \frac{N_1}{N_2}$$

a) If you change label of i, change sign of i

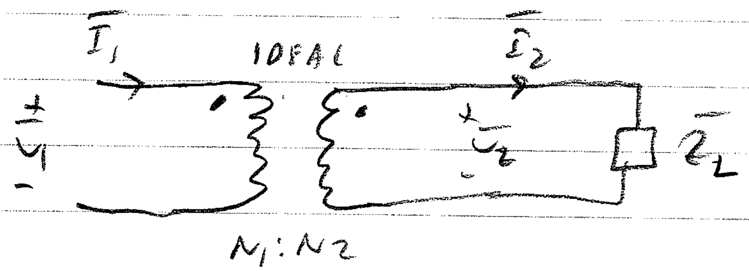
b) If " " " " v, " " " v

c) If " " the dots, change sign of i & v

Can show this means $k = 1$

$L_1 = L_2 = M = \infty$

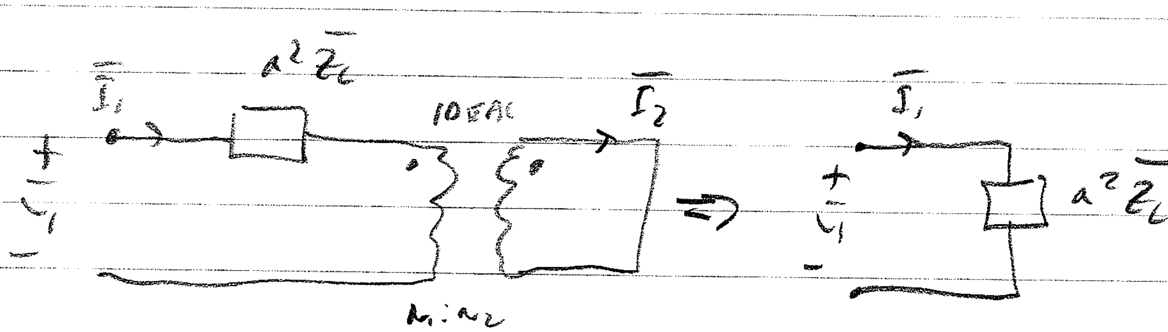
"Refer" impedances across transformer



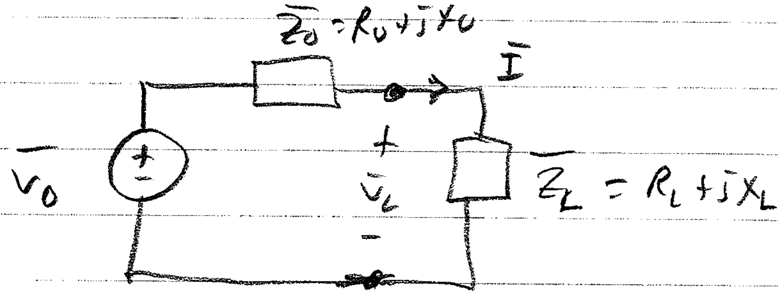
$$\bar{V}_2 = \bar{I}_2 \bar{Z}_L$$

$$\bar{V}_1 = \frac{N_1}{N_2}\bar{V}_2 = \frac{N_1}{N_2}\left(\bar{I}_2 \bar{Z}_L\right) = \frac{N_1}{N_2}\left(\left(\frac{N_1}{N_2}\bar{I}_1\right)\bar{Z}_L\right)$$

$$= \left(\frac{N_1}{N_2}\right)^2 \bar{Z}_L \bar{I}_1$$



Maximum Power Transfer



$P_{\substack{TO\\load}} = \text{Real}(\bar{V}_L \bar{I}^*)$ **OR**

$$= |\bar{I}|^2 R_L = \frac{|\bar{V}_0|^2 R_L}{\left((R_0+R_L)^2 + (X_0+X_L)^2\right)}$$

For fixed $|\bar{V}_0|$, R_0, X_0, what should R_L & X_L be?

maximize $P_{\substack{TO\\load}}$ with respect to R_L & X_L

By inspection, set $\boxed{X_L = -X_0}$

$$P_{\substack{TO\\load\\X_L=-X_0}} = \frac{|\bar{V}_0|^2 R_L}{(R_0+R_L)^2}$$

$$\frac{\partial P_{\substack{TO\\ Load}}}{\partial R_L} = |\bar{V}_0|^2 \left(\frac{1}{(R_0+R_L)^2} - \frac{2R_L}{(R_0+R_L)^3} \right) = 0$$

multiply by $(R_0+R_L)^2$ get $(R_0+R_L) = 2R_L$

$$\boxed{R_L = R_0}$$

OR $\boxed{\bar{Z}_L = \bar{Z}_0^*}$

what if $X_L = 0$ (no reactance) ?

$$P_{\substack{TO\\ load}} = \frac{|\bar{V}_0|^2 R_L}{(R_0+R_L)^2 + X_0^2}$$

$$\frac{\partial P_{\substack{TO\\ load}}}{\partial R_L} = |\bar{V}_0|^2 \left(\frac{1}{(R_0+R_L)^2+X_0^2} - \frac{R_L\left(2(R_0+R_L)\right)}{\left((R_0+R_L)^2+X_0^2\right)^2} \right) = 0$$

multiply by $\left((R_0+R_L)^2+X_0^2\right)$ get

$$(R_0+R_L)^2 + X_0^2 = 2R_L(R_0+R_L)$$

$$R_0^2 + \cancel{2R_0R_L} + R_L^2 + X_0^2 = \cancel{2R_LR_0} + 2R_L^2$$

$$R_L^2 = R_0^2 + X_0^2 \qquad \boxed{R_L = \sqrt{R_0^2+X_0^2} = |\bar{Z}_0|}$$

What if X_L cannot be changed?

$$P_{\substack{TO\\load}} = \frac{|\bar{V}_0|^2 R_L}{(R_0+R_L)^2+(X_0+X_L)^2}$$

For fixed X_L

$$\frac{\partial P_{\substack{TO\\load}}}{\partial R_L} = |\bar{V}_0|^2\left(\frac{1}{(R_0+R_L)^2+(X_0+X_L)^2} - \frac{2R_L(R_0+R_L)}{\left((R_0+R_L)^2+(X_0+X_L)^2\right)^2}\right)$$

$$(R_0+R_L)^2+(X_0+X_L)^2 = 2R_LR_0+2R_L^2$$

$$R_0^2+2\cancel{R_0R_L}+R_L^2+(X_0+X_L)^2 = 2\cancel{R_LR_0}+2R_L^2$$

$$R_L^2 = R_0^2+(X_0+X_L)^2$$

$$R_L = \sqrt{R_0^2+(X_0+X_L)^2}$$

what if R_L and X_L are constrained so

$$\frac{R_L}{X_L} = \text{constant?} \qquad (\text{i.e. } X_L = c R_L)$$

$$P_{\substack{TO\\LOAD}} = \frac{|\bar{V}_0|^2 R_L}{(R_0+R_L)^2 + (X_0 + cR_L)^2}$$

$$\frac{\partial P_{\substack{TO\\LOAD}}}{\partial R_L} = |\bar{V}_0|^2 \left(\frac{1}{(R_0+R_L)^2 + (X_0+cR_L)^2} - \frac{R_L\left(2(R_0+R_L) + 2c(X_0+cR_L)\right)}{\left((R_0+R_L)^2 + (X_0+cR_L)^2\right)^2} \right)$$

$$= 0$$

get $\quad (R_0+R_L)^2 + (X_0+cR_L)^2 = \cancel{2R_0R_L} + 2R_L^2 + \cancel{2cR_LX_0} + 2c^2R_L^2$

$$R_0^2 + \cancel{2R_0R_L} + R_L^2 + X_0^2 + \cancel{2cR_LX_0} + c^2R_L^2 = \quad ''$$

$$R_0^2 + X_0^2 = c^2R_L^2 + R_L^2 = R_L^2(1+c^2)$$

$$R_L = \frac{1}{\sqrt{1+c^2}}\sqrt{R_0^2+X_0^2}$$

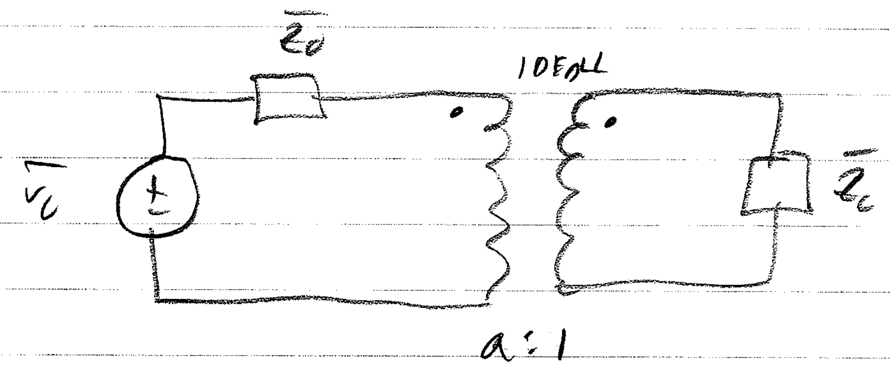
$$X_L = \frac{c}{\sqrt{1+c^2}}\sqrt{R_0^2+X_0^2}$$

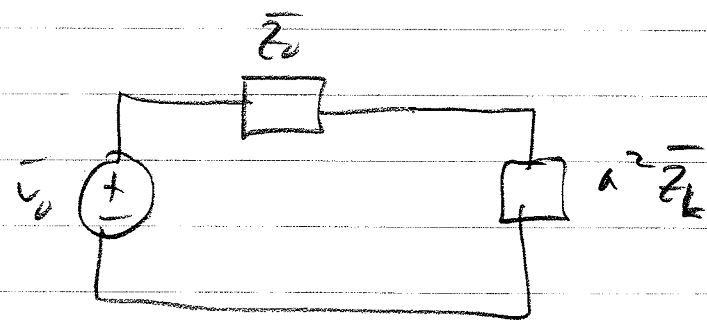
OR $\quad R_L^2 + X_L^2 = R_0^2 + X_0^2$

$$|\bar{Z}_L| = |\bar{Z}_0|$$

Use this to maximize power to fixed $\bar{Z}_L$

using a transformer





Select a so that $\left|a^2 \bar{Z}_L\right| = \left|\bar{Z}_0\right|$

$$a = \sqrt{\frac{|\bar{Z}_0|}{|\bar{Z}_L|}}$$

#15

Power Transformer

Design consideration #1 => flux level

$v_1 = V_{m_1} \cos \omega t \quad = N_1 \frac{d\phi}{dt}$

$\phi = \frac{V_{m_1} \sin \omega t}{N_1 \omega} \qquad \phi_{max} = \frac{V_{m_1}}{2\pi f N_1}$

$V_{m_1} = 2\pi f N_1 \phi_{max}$

$V_{Rms} = \sqrt{2}\pi f N_1 \phi_{max}$

$= 4.44 f N_1 \phi_{max}$

Given Specific $V_{Rms} = 230$ and $f = 60$ Hz

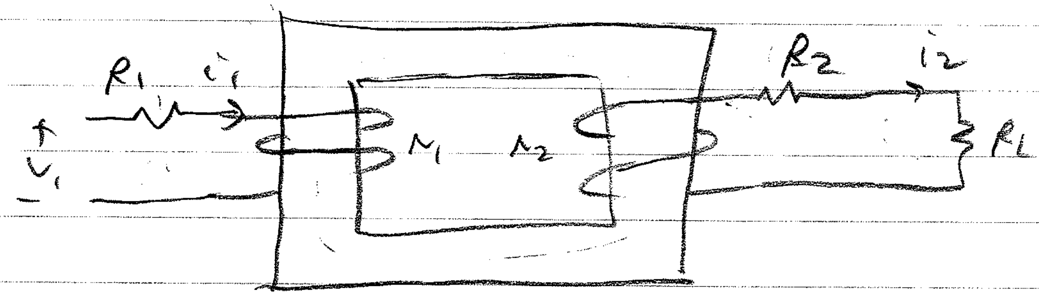
$\phi_{max} = \frac{230}{4.44 \times 60 N_1} \qquad B_{max} = \frac{\phi_{max}}{Area}$

Given $N_1 = 200 \quad A_{rea} = .005\ m^2$

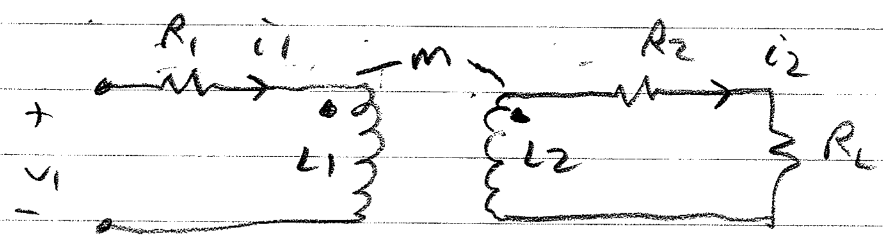
$B_{max} = 0.864\ T$

Now add winding Resistance, leakage L & magnetizing Reac.

Recall



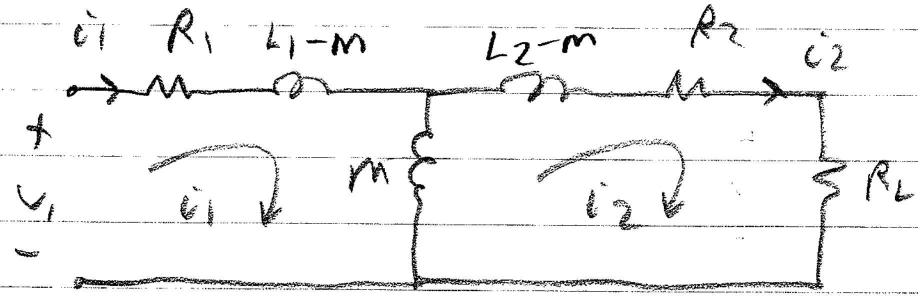
⇓



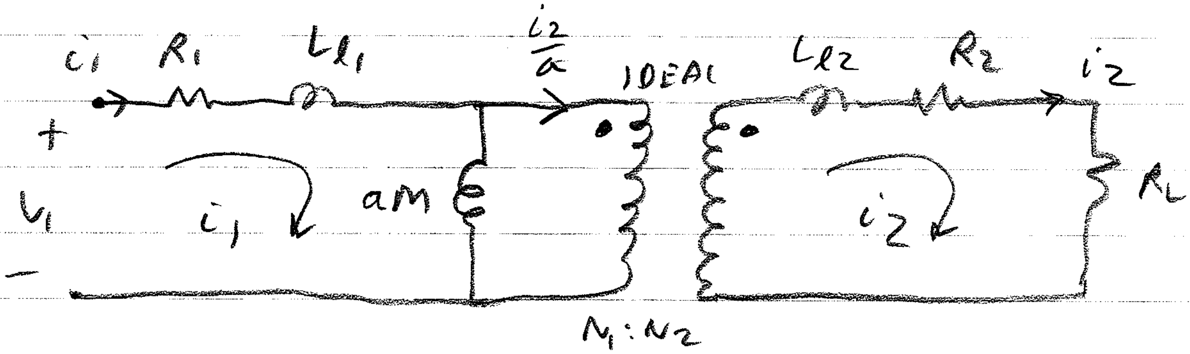
$$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$0 = i_2 R_2 + i_2 R_L + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

⇓



OR



$$a = \frac{N_1}{N_2}$$

$$v_1 = R_1 i_1 + L_{\ell_1} \frac{di_1}{dt} + aM\left(\frac{di_1}{dt} - \frac{1}{a}\frac{di_2}{dt}\right)$$

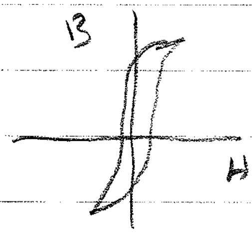
$$\cancel{a}M\left(\frac{di_1}{dt} - \frac{1}{a}\frac{di_2}{dt}\right) = \cancel{a}\left[L_{\ell_2}\frac{di_2}{dt} + R_2 i_2 + R_L i_2\right]$$

$$\left(\begin{array}{lcl} L_{\ell_1} + aM = L_1 & \Rightarrow & L_{\ell_1} = L_1 - aM \\ L_{\ell_2} + \frac{M}{a} = L_2 & & L_{\ell_2} = L_2 - \frac{M}{a} \end{array}\right)$$

#16

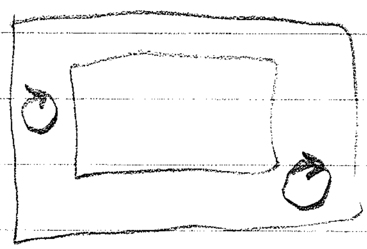
Core losses

1. Hysteresis $P_h \approx K_h f B_m^{1.6}$ (Heinstiz)



realign magnetic domains

2. Eddy currents



circulating currents in iron
Laminations reduce this in the "depth" direction

$$P_e \approx K_e f^2 B_m^2$$

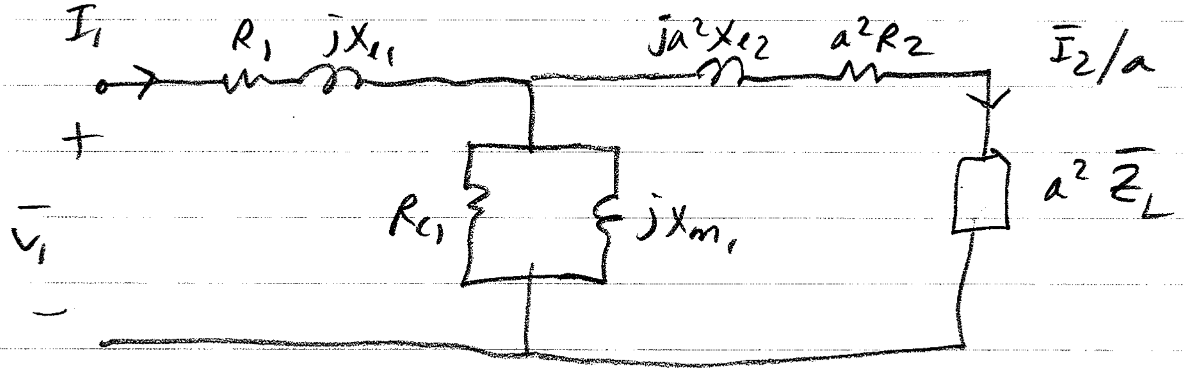
Since B_m depends on voltage,

$$B_m \propto V_{source}$$

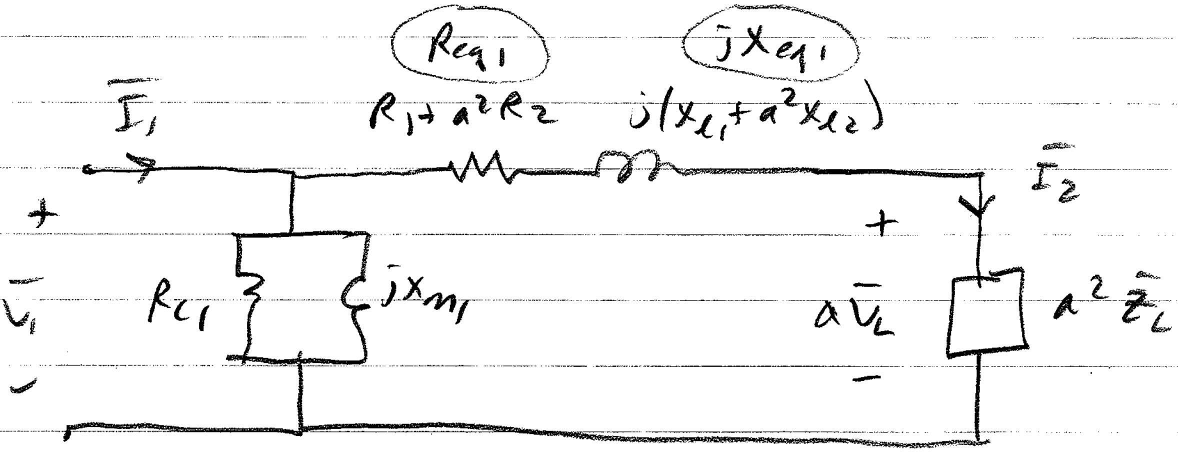
Add as shunt resistor

Sinusoidal Steady State (Phasors etc.)

(and "refer" to 1 side)



Approximate equivalent circuit



Efficiency $\eta = \frac{P_{out}}{P_{in}} \times 100$

Voltage regulation $VR = \frac{|\bar{V}_{L_{\text{no load}}}| - |\bar{V}_{L_{\text{load}}}|}{|\bar{V}_{L_{\text{load}}}|} \times 100$

$$\left(\begin{array}{c} \text{SC Test} \\ \text{OC Test} \end{array} \right)$$

<u>Nameplate</u>

Rated V_1/V_2

Rated KVA

$\% Z$

$$\% Z \triangleq \frac{\sqrt{R_{eq_1}^2 + X_{eq_1}^2}}{Z_{rated}} \times 100$$

$$Z_{rated} = \frac{V_{1rated}}{I_{1rated}} = \frac{V_{1rated}}{S_{rated}/V_{1rated}} = \frac{V_{1rated}^2}{S_{rated}}$$

Can show if $|\bar{V}_1| = V_{1rated}$ and secondary is shorted,

$$I_{sc} = \frac{100}{\% Z} \times I_{rated}$$