Polarity marks (the dot convention)

The dots are only needed when there are two or more coils.

The dots tell you how the coil is "wound".

TO find the second dot:
1. Inject current into "first dot" and use Right Hand Rule to find flux direction in iron.

i.e.  

\[ A \rightarrow B \]

\[ C \rightarrow D \]
2. Put "second dot" on the terminal of the other coil whose injection current gives the same flux direction of iron.

i.e. "Second dot" goes on C because if you inject current into C, it will create ϕ in same direction.

Example

1. Pick terminal A for first dot
2. Inject current into A flux goes → through iron of top coil and ← through bottom coil.
3. To get flux due to bottom coil to go ← inject current into C: so dot on C
Writing equations

\[ V_1 = i_1 R_1 + \frac{dL_1}{dt} \]
\[ V_2 = i_2 R_2 + \frac{dL_2}{dt} \]

with resistance included,

Magnetic analysis would show that

\[ \mathcal{A}_1 = L_1 i_1 - m i_2 \] \[ \mathcal{A}_2 = -m i_1 + L_2 i_2 \]

so:
\[ V_1 = i_1 R_1 + L_1 \frac{di_1}{dt} - m \frac{di_2}{dt} \]
\[ V_2 = i_2 R_2 - m \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \]
The "circuit model" for this is:

\[ V_1 = L_1 R_1 + L_1 \frac{dl_1}{dt} - M \frac{dl_2}{dt} \]

Nothing to do with dots minus because we are summing drops in left KVL and we want drop from "dot" to "under dot"

So:

The mutual term will be \( M \) times the derivative of the current entering the dot of coil 2. This is \(-i_2\)
Example

\[ \begin{align*}
-\dot{V} + R_1 \dot{i} + L_1 \frac{\dot{i}}{dt} &= -m \frac{\dot{i}}{dt} + R_2 \dot{i} + L_2 \frac{\dot{i}_2}{dt} - m \frac{\dot{i}}{dt} \\
\text{do not need dots for this} & \quad \text{do not use dots for this}
\end{align*} \]

\[ \begin{align*}
\text{minus because we want voltage drop from dot to undot of coil 1, so we use the current} \\
\text{into the dot of coil 2 - which is } -\dot{i} & \quad \text{into the undot of coil 1 - which is } -\dot{i}
\end{align*} \]

Go through Example 2.6 if time allows
1. Derive ideal model

\[ \sigma = \infty \]

\[ \mu = \infty \]

\[ N_1 i_1 = N_2 i_2 \]

\[ V_2 = \frac{N_2}{N_1} V_1 \]

\[ a = \frac{N_1}{N_2} \]

a) If you change label of \( i \) change sign of \( i \).

b) \[ \cdots \]

c) \[ \cdots \]

Can show this means \( b = 1 \)

\[ l_1 = l_2 = M = \infty \]
Refer impedances across transformer

\[ \overline{V}_t = \overline{E}_2 \overline{Z}_L \]

\[ \overline{V}_1 = \frac{N_1}{N_2} \overline{V}_2 = \frac{N_1}{N_2} (\overline{E}_2 \overline{Z}_L) = \frac{N_1}{N_2} \left( \frac{N_1}{N_2} \overline{E}_1 \right) \overline{Z}_L \]

\[ = \left( \frac{N_1}{N_2} \right)^2 \overline{Z}_L \overline{I}_1 \]

\[ \overline{E}_1 \]

\[ \overline{Z}_L \]

\[ \overline{E}_2 \]

\[ \overline{I}_1 \]

\[ \overline{I}_2 \]

\[ \overline{Z}_L \]

\[ N_1 : N_2 \]
Maximum Power Transfer

\[ \overline{E_0} = R_0 + jx_0 \]

\[ V_0 \]

\[ Z_L = R_L + jx_L \]

\[ P_{to} = \text{Real} \left( \overline{V_0} \overline{I} \right)^* \] OR

\[ = |\overline{I}|^2 R_L = \frac{|V_0|^2 R_L}{\left( (R_0+R_L)^2 + (x_0+x_L)^2 \right)^2} \]

For fixed \( |V_0|, R_0, x_0 \), what should \( R_L \& x_L \) be?

\[ \text{maximize } P_{to} \text{ with respect to } R_L \& x_L \text{ load} \]

By inspection, set \[ x_L = -x_0 \]

\[ P_{to} = \frac{|V_0|^2 R_L}{(R_0+R_L)^2} \text{ when } x_L = -x_0 \]
\[
\frac{2P_{0}}{Z_{L}} = \left| V_{0} \right|^{2} \left( \frac{1}{(R_{0}+R_{L})^2} - \frac{2R_{L}}{(R_{0}+R_{L})^3} \right) = 0
\]

Multiply both sides by \((R_{0}+R_{L})^2\) and get \((R_{0}+R_{L}) = 2R_{L}\)

\[R_{L} = R_{0}\]

OR

\[Z_{L} = Z_{0}^{*}\]

\[\text{What if } X_{L} = 0 \text{ (no reactance)}?\]

\[
P_{T0} = \frac{1}{\text{load}} \cdot \frac{|V_{0}|^2 R_{L}}{(R_{0}+R_{L})^2 + X_{0}^2}
\]

\[
\frac{2P_{T0}}{Z_{L}} = \left| V_{0} \right|^{2} \left( \frac{1}{(R_{0}+R_{L})^2 + X_{0}^2} - \frac{R_{L}}{(R_{0}+R_{L})^3 + X_{0}^2} \right) = 0
\]

Multiply both sides by \((R_{0}+R_{L})^2 + X_{0}^2\) and get

\[(R_{0}+R_{L})^2 + X_{0}^2 = 2R_{L}(R_{0}+R_{L})\]

\[R_{0}^2 + 2R_{0}R_{L} + R_{L}^2 + X_{0}^2 = 2R_{L}R_{0} + 2R_{L}^2\]

\[R_{L}^2 = R_{0}^2 + X_{0}^2, \quad R_{L} = \sqrt{R_{0}^2 + X_{0}^2} = |Z_{0}|\]
What if $X_L$ cannot be changed?

\[ P_T = \left(\frac{1}{2}\right)^2 P_L \]

\[ \frac{P_T}{10^{30}} = \frac{1}{(R_L + R_0)^2 + (x_0 + x_L)^2} \]

\[ \mu_{\omega} f_{\omega} d X_L \]

\[ \frac{2P_T}{10^{30}} = \left(\frac{1}{(R_L + R_0)^2 + (x_0 + x_L)^2} \right) - \frac{2R_L (R_0 + R_L)}{(R_0 + R_L)^2 + (x_0 + x_L)^2} \]

\[ (R_0 + R_L)^2 + (x_0 + x_L)^2 = 2R_0 R_L + 2R_L^2 \]

\[ R_0^2 + 2R_0 R_L + R_L^2 + (x_0 + x_L)^2 = 2R_0 R_L + 2R_L^2 \]

\[ R_L^2 = R_0^2 + (x_0 + x_L)^2 \]

\[ R_L = \sqrt{R_0^2 + (x_0 + x_L)^2} \]
What if $R_L$ and $X_L$ are constrained so

$$\frac{R_L}{X_L} = \text{constant} \Rightarrow (i.e. \ X_L = CR_L)$$

$$P_{to} = \frac{|V_0|^2 R_L}{(R_0 + R_L)^2 + (X_0 + CR_L)^2}$$

$$\frac{\partial P_{to}}{\partial R_L} = |V_0|^2 \left( \frac{1}{(R_0 + R_L)^2 + (X_0 + CR_L)^2} - \frac{R_L \left( 2(R_0 + R_L) + 2C(X_0 + CR_L) \right)}{(R_0 + R_L)^2 + (X_0 + CR_L)^2} \right)^2 = 0$$

get $$(R_0 + R_L)^2 + (X_0 + CR_L)^2 = \frac{2R_0 R_L + 2R_L^2}{2C^2 R_L^2 + 2C^2 X_0^2 + 2C^2 L^2}$$

$$R_0^2 + 2R_0R_L + R_L^2 + X_0^2 + 2C^2 X_0 R_L + 2C^2 L^2 = \frac{11}{\sqrt{1 + C^2}}$$

$$R_0^2 + X_0^2 = \frac{11}{\sqrt{1 + C^2}}$$

$$R_L = \frac{1}{\sqrt{1 + C^2}} \sqrt{R_0^2 + X_0^2}$$

$$X_L = \frac{C}{\sqrt{1 + C^2}} \sqrt{R_0^2 + X_0^2}$$

OK, $$R_L^2 + X_L^2 = R_0^2 + X_0^2$$

$$|Z_L| = 1.7Z_0$$
Use this to maximize power to fixed $\bar{Z}_L$

Using a transformer

$$a : 1$$

Select $a$ so that

$$|a^2 \bar{Z}_L| = |\bar{Z}_L|$$

$$a = \sqrt{\frac{|\bar{Z}_L|}{|\bar{Z}_L|}}$$
Power Transformer

Design consideration #1 = flux level

\[ N_1 = U_m \cos \omega t = N_1 \frac{d\phi}{dt} \]

\[ \phi = \frac{U_m \sin \omega t}{N_1} \quad \phi_{\text{max}} = \frac{U_m}{2\pi f N_1} \]

\[ U_m = 2\pi f N_1 \phi_{\text{max}} \]

\[ V_{\text{rms}} = \sqrt{2} \pi f N_1 \phi_{\text{max}} \]

\[ = 4.44 f N_1 \phi_{\text{max}} \]

Given specification: \( V_{\text{rms}} = 230 \) mV, \( f = 60 \) Hz

\[ \phi_{\text{max}} = \frac{230}{4.44 \times 60 N_1} \quad \beta_{\text{max}} = \frac{\phi_{\text{max}}}{\alpha \cdot a} \]

Given \( N_1 = 200 \) \( A_{\text{tan}} = 1.005 \) m²

\[ \beta_{\text{max}} = 0.864 T \]
Now add winding resistance, leakage $L$ & magnetomotive force

Recall

\[ v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} - m \frac{di_2}{dt} \]

\[ 0 = i_2 R_2 + i_2 R_L + L_2 \frac{di_2}{dt} - m \frac{di_1}{dt} \]
\[ N_1 = R_1 i_1 + L_{i_1} \frac{di_1}{dt} + aM\left(\frac{di_1}{dt} - \frac{1}{a}\frac{di_2}{dt}\right) \]

\[ aM\left(\frac{di_1}{dt} - \frac{1}{a}\frac{di_2}{dt}\right) = \kappa \left[ L_{i_2} \frac{di_2}{dt} + R_2 i_2 + R_L i_2 \right] \]

\[
\begin{align*}
L_{i_1} + aM &= L_1 \\
L_{i_2} + \frac{M}{a} &= L_2
\end{align*}
\]

\[
\begin{align*}
L_{i_1} &= L_1 - aM \\
L_{i_2} &= L_2 - \frac{M}{a}
\end{align*}
\]
Core losses

1. Hysteresis \( P_h \propto k_h f B_m^{1.6} \) (Hunt 74)
   \[ \int \vec{B} \cdot d\vec{H} \]
   realign magnetic domain

2. Eddy current
   circulating currents in iron
   Laminations reduce this in the "depth" direction
   \[ P_e = k_e f^2 B_m^2 \]

Since \( B_m \) depends on voltage,
   \[ B_m \propto V_{\text{source}} \]

Add as shunt resistor
Sinusoidal steady state (Phasor etc.)

(approximate equivalent circuit)

Efficiency \( \eta = \frac{P_{out}}{P_{in}} \times 100 \)

Voltage regulation \( VR = \left( \frac{|V_L|}{{\mu V_{100\%}}} - \frac{|V_L|}{{\mu V_{10\%}}} \right) \times 100 \)
Rated $V_1/V_2$

Rated $kVA$

$\% Z = \sqrt{R_{eq1}^2 + X_{eq1}^2} \times 100 \over Z_{rated}$

$Z_{rated} = \frac{V_{rated}}{I_{rated}} = \frac{V_{rated}}{I_{rated}/V_{rated}} = \frac{V_{rated}^2}{S_{rated}}$

Can show if $|V_1| = V_{rated}$ and Second is Shunted,

$\frac{I_{sc}}{I_{rated}} = \frac{100}{\% Z} \times I_{rated}$