#1

1.2 Physical Structures

The major components of a power system can be represented in a block-diagram format, as shown in Figure 1.1.

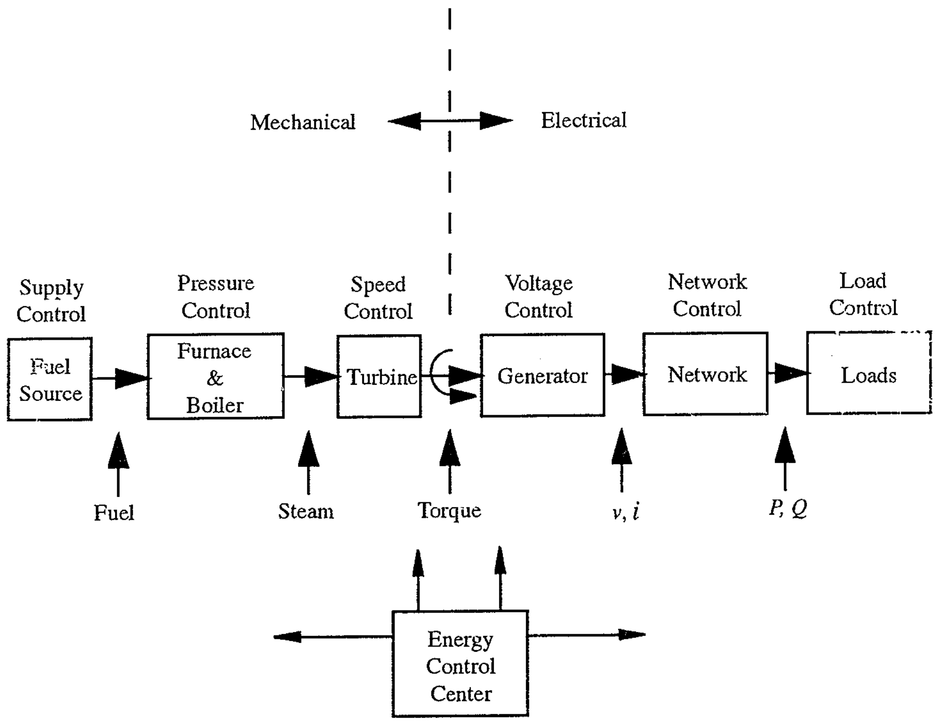


Figure 1.1: *System dynamic structure*

While this block diagram representation does not show all of the complex dynamic interaction between components and their controls, it serves to broadly describe the dynamic structures involved. Historically, there has been a major division into the mechanical and electrical subsystems as shown. This division is not absolute, however, since the electrical side clearly contains components with mechanical dynamics (tap-changing-under-load (TCUL) transformers, motor loads, etc.) and the mechanical side clearly contains components with electrical dynamics (auxiliary motor drives, process controls, etc.). Furthermore, both sides are coupled through the monitoring and control functions of the energy control center.

ECE 333

#1 Go through outline & Course Guide

Power & 3ϕ

Review Phasors

Consider $v(t) = V_m \cos(\omega t + \theta_v)$

Euler's expansion $e^{j\theta} = \cos\theta + j\sin\theta$

So: $v(t) = \mathrm{Real}\left(V_m e^{j(\omega t + \theta_v)}\right) = \mathrm{Real}\left(V_m e^{j\theta_v} e^{j\omega t}\right)$

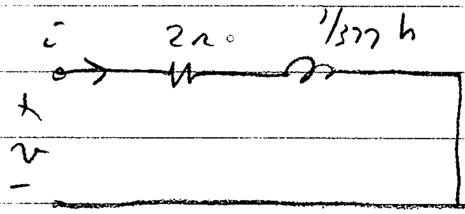
$= \mathrm{Real}\left(\sqrt{2}\,\bar{V} e^{j\omega t}\right)$

$\bar{V} \overset{\Delta}{=} \dfrac{V_m}{\sqrt{2}} e^{j\theta_v}$ (RMS PHASOR - COSINE REF)

(Recall $F_{RMS} = \sqrt{\dfrac{1}{T}\displaystyle\int_{t_0}^{t_0+T} f^2(t)\,dt}$ (ROOT OF MEAN OF SQUARE))

STOP #1

Example

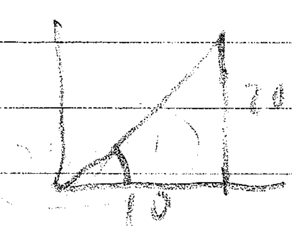


For $i = 10 \sin 377t$ A, find v

$v = iR + L\dfrac{di}{dt} = 20\sin 377t + \frac{1}{377}(377 \times 10 \cos 377t)$

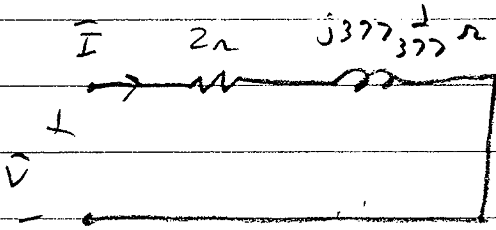
$= 20\sin 377t + 10\cos 377t = \sqrt{20^2 + 10^2}\cos\left(377t - \tan^{-1}\frac{20}{10}\right)$

$= \underline{22.36\cos(377t - 63.4°)\ V}$



#2

OR, use frequency (phasor) domain



$\bar{I} = 7.07\angle -90^\circ$ A. Rms

$\bar{V} = (2+j1)7.07\angle -90 = \sqrt{5}\angle 26.6^\circ \; 7.07\angle -90^\circ = 15.8\angle -63.4^\circ$ V. Rms

$v(t) = 22.36\cos(377t - 63.4^\circ)$ V.

Recall $\quad v(t) \rightarrow \bar{V} \qquad R \rightarrow R$

$i(t) \rightarrow \bar{I} \qquad L \rightarrow j\omega L = jX_L \; (X_L = \omega L)$

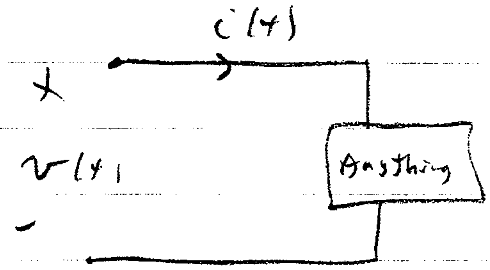
$C \rightarrow \frac{1}{j\omega C} = jX_C \; (X_C = -\frac{1}{\omega C})$

Series R, L $\quad \bar{Z} = R + j\omega L$

series R, C $\quad \bar{Z} = R - j\frac{1}{\omega C}$

series R, L, C $\quad \bar{Z} = R + j(\omega L - \frac{1}{\omega C})$

$\bar{V} = \bar{Z}\bar{I}$ $\quad$ ($\bar{Z}$ is not a phasor)



1. $p_{in}(t) = v(t)\,i(t)$ watts instantaneous power

2. If $v(t) = V_m \cos(\omega t + \theta_v)$
   $i(t) = I_m \cos(\omega t + \theta_i)$

Then $p_{in}(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$

$$= V_m I_m \left\{ \tfrac{1}{2}\cos\left[(\omega t+\theta_v) - (\omega t + \theta_i)\right] + \tfrac{1}{2}\cos\left[(\omega t + \theta_v) + (\omega t + \theta_i)\right] \right\}$$

$$= \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v + \theta_i)$$

2009
2004
+
2005

3. $P_{in} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$ (Ave or real power) watts

4. $PF = \cos(\theta_v - \theta_i)$ power factor of P_{in}
   lagging if $0 < \theta_v - \theta_i < 180$
   leading if $-180 < \theta_v - \theta_i < 0$

5. $P_{app} = \frac{V_m I_m}{2}$ (Apparent power) voltamps

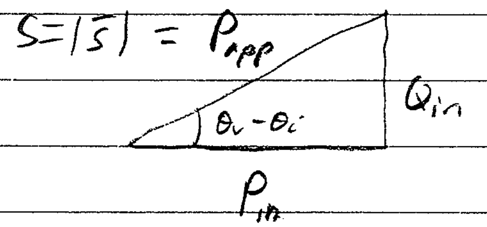
STOP 2006

$$p(t) = P(1+\cos(2\omega t + 2\theta_i)) - Q\sin(2\omega t + 2\theta_i)$$

6. $Q_{in} = \dfrac{V_m I_m}{2} \sin(\theta_v - \theta_i)$ (reactive power) vars

7. $\bar{S}_{in} = P_{in} + jQ_{in}$ (complex power) voltamps

$= P_{app} \angle \theta_v - \theta_i$

8. 

$S = |\bar{S}_{in}| = \sqrt{P_{in}^2 + Q_{in}^2}$

9. Note, for $\bar{V}$ $\bar{I}$ rms phasors,

$$\bar{S} = \bar{V}\bar{I}^* = \frac{V_m}{\sqrt{2}}\angle\theta_v \, \frac{I_m}{\sqrt{2}}\angle -\theta_i = \frac{V_m I_m}{2}\angle \theta_v - \theta_i \checkmark$$

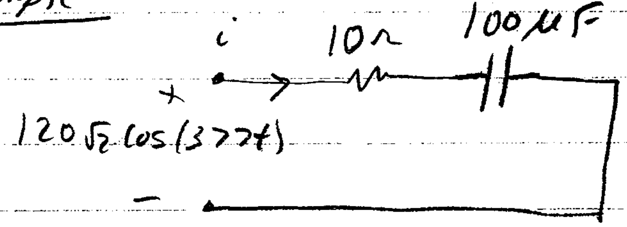
10. note also, $\bar{V} = \bar{Z}\bar{I}$

$\bar{Z} = \dfrac{\bar{V}}{\bar{I}}$ $\quad \angle\bar{Z} = \angle\bar{V} - \angle\bar{I} = \theta_v - \theta_i$

so: angle on $\bar{Z}$ equals angle on $\bar{S}$

for $\bar{Z}$, pf = cos(angle on $\bar{Z}$)

Example



Find: i_{ss} $\bar{S}_{IN}$ P_{IN} Q_{IN} PF

Soln: $\bar{V} = 120\angle 0$ $\quad \bar{Z} = 10 - j26.5 = 28.3\angle -69^\circ$

$$\bar{I} = \frac{120\angle 0}{28.3\angle -69^\circ} = 4.24\angle +69^\circ$$

$$i_{ss} = 4.24\sqrt{2}\cos(377t + 69^\circ)$$

$$\bar{S}_{IN} = \bar{V}\bar{I}^* = 120\angle 0 \; 4.24\angle -69^\circ = 509\angle -69^\circ \text{ voltamps}$$

$$P_{IN} = 509 \cos -69^\circ = 182 \text{ W}$$

$$Q_{IN} = 509 \sin -69^\circ = -475 \text{ vars}$$

Note $P^2 + Q^2 = |\bar{S}|^2$

$$PF = \cos -69^\circ = 0.34 \text{ lead} \quad (\theta_v - \theta_i = -69^\circ)$$

Work Series Parallel examples

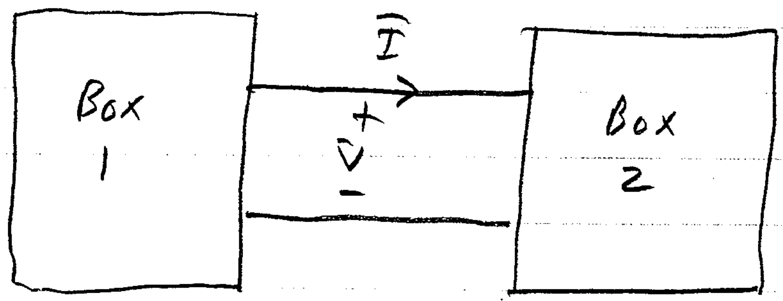
Add this to text

#3

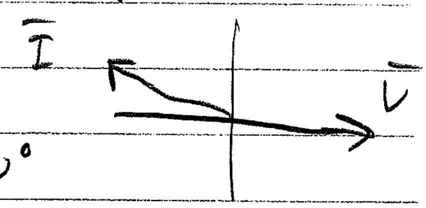
(1) $\bar{V}$ & $\bar{I}$ are phasors

$\bar{S}$ & $\bar{Z}$ are <u>not</u> phasors

(2)



We have "assumed" box 2 is the "load"



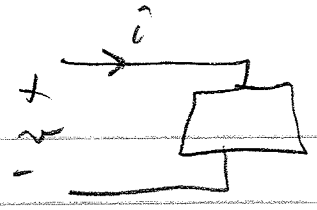
suppose

$$\bar{V} = 120\angle 0 \qquad \bar{I} = 4\angle 150^\circ$$

$$\underset{\text{Box 2}}{\bar{S}_{IN}} = 480\angle -150^\circ = \underset{\text{Box 1}}{\bar{S}_{OUT}}$$ PF is not defined (Box 2 is source)

$$\underset{\text{Box 2}}{\bar{S}_{OUT}} = -480\angle -150^\circ = 480\angle 30^\circ$$ PF = .866 Lag

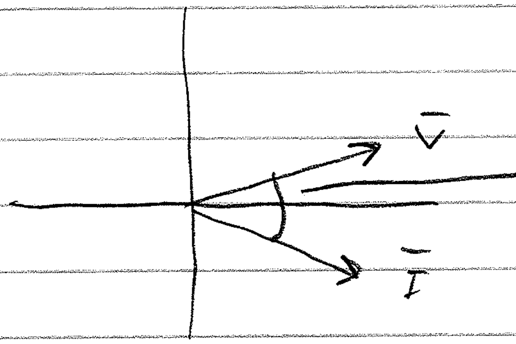
$$\underset{\text{Box}_1}{\bar{S}_{IN}} = \underset{\text{Box 2}}{\bar{S}_{OUT}}$$



Sketch phasors

$v(t) = 10\sqrt{2}\cos(\omega t + 30°)$ $\qquad \bar{V} = 10\angle 30°$

$i(t) = 5\sqrt{2}\sin(\omega t + 70°)$ $\qquad \bar{I} = 5\angle -20°$



PF Angle $= \theta_v - \theta_i$
$= 30 - (-20)$
$= 50°$

$\bar{S}_{IN} = \bar{V}\bar{I}^* = 10\angle 30 \; 5\angle 20 = 50\angle 50°$

$= (50\cos 50 + j50\sin 50)$

$= 32.14 + j38.3 \qquad = P_{IN} + jQ_{IN}$

How many VARs do you need to "add" to make it unity power factor?

2003 Ans: 38.3

How many VARs do you need to "add" to make PF = .9 lag?

$S = \frac{32.14}{.9} = 35.71 \qquad \theta = \cos^{-1}.9 = 25.84 \qquad 35.715\sin 25.84 = 15.56$ ANS 22.7

Series

Parallel

Power triangle

Specifying Power of load

S (kVA) + power factor

S (kVA) + P (watts)

S (kVA) + Q (VArs)

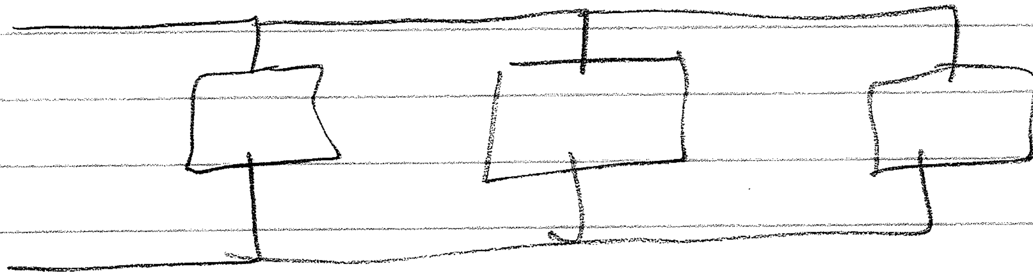
P (watts) + Q (VArs)

$\bar{V}$, $\bar{I}$ (note can assume angle on I)

$\bar{V}$, $\bar{Z}$

$\bar{I}$, $\bar{Z}$

Problem 2.11



250 kVA .5 PF lag   180 kW .8 lead   283 + j100 kVA

$$\bar{S}_{TOT} = 250k \underline{/\cos^{-1}.5} + \frac{180k}{.8} \underline{/-\cos^{-1}.8} + (283 + j100)k$$

$$= 125k + j216.5k + 180k - j135^k + 283^k + j100k$$

$$= 588k + j181.5k$$

$$= 615k \underline{/17.15^\circ} \qquad \underline{PF = .9555 \text{ lag}}$$

Find Q to make PF = 0.8 lead (not sure why)

$$\bar{S} = 588k + j181.5k + jQ_{add}$$

want $\bar{S} = \frac{588k}{.8} \underline{/-\cos^{-1}.8} = 735k \underline{/-37^\circ} = 588 - j442\ K$

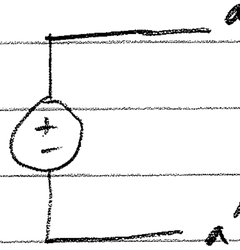
$181.5k + Q_{add} = -442 \qquad Q_{add} = -182 - 442$

Stop #3

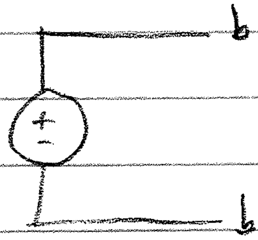
Do 1-phase residentize first

#4

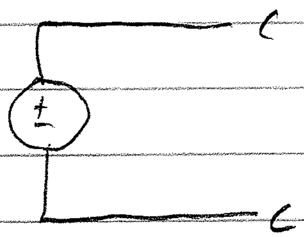
Three phase



$$v_{aa'} = V_m \cos(\omega t + \theta_v)$$



$$v_{bb'} = V_m \cos(\omega t + \theta_v - 120^\circ)$$



$$v_{cc'} = V_m \cos(\omega t + \theta_v + 120^\circ)$$

This is called positive (abc) sequence

An alternative would be negative (acb) sequence

- change (-120°) to $(+120^\circ)$ on b
- change $(+120^\circ)$ to (-120°) on c

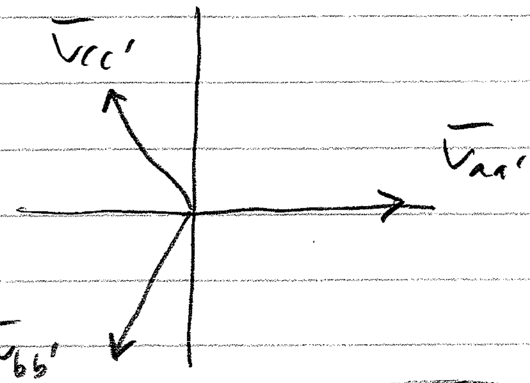
abc

$$\bar{V}_{aa'} = \frac{V_m}{\sqrt{2}} \angle \theta_v$$

$$\bar{V}_{bb'} = \frac{V_m}{\sqrt{2}} \angle \theta_v - 120^\circ$$

$$\bar{V}_{cc'} = \frac{V_m}{\sqrt{2}} \angle \theta_v + 120^\circ$$

Let $\theta_v = 0$



Note: $\bar{V}_{aa'} + \bar{V}_{bb'} + \bar{V}_{cc'} = 0$

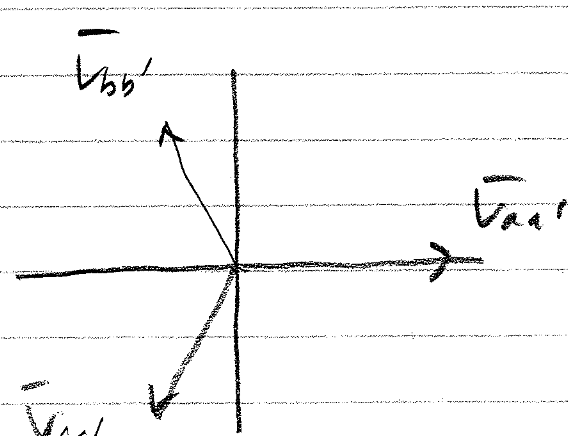
acb

$$\bar{V}_{aa'} = \frac{V_m}{\sqrt{2}} \angle \theta_v$$

$$\bar{V}_{bb'} = \frac{V_m}{\sqrt{2}} \angle \theta_v + 120^\circ$$

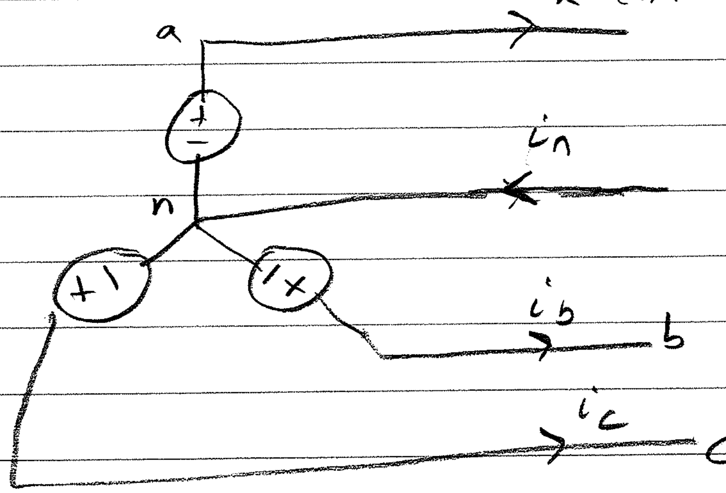
$$\bar{V}_{cc'} = \frac{V_m}{\sqrt{2}} \angle \theta_v - 120^\circ$$

Let $\theta_v = 0$



3-phase connections

4-wire WYE ($Y_{\perp}$) Source (connect primes together)



(line to neutral voltage equals phase voltage V_ϕ)

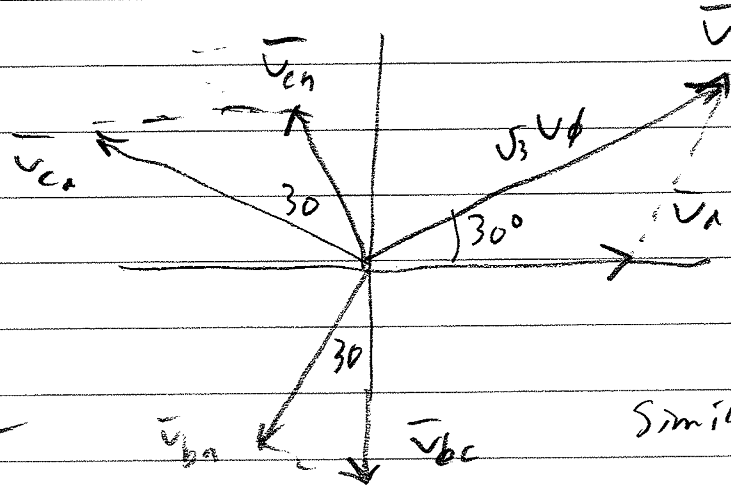
("line" voltage = voltage between 2 lines

i.e. $v_{ab} = v_{an} + v_{nb} = v_{an} - v_{bn}$)

For ABC sequence ($\theta_v = 0$)

$$\bar{V}_{ab} = V_\phi \angle 0 - V_\phi \angle -120^\circ$$

$\bar{V}_{ab} = \sqrt{3} V_\phi \angle 30^\circ$

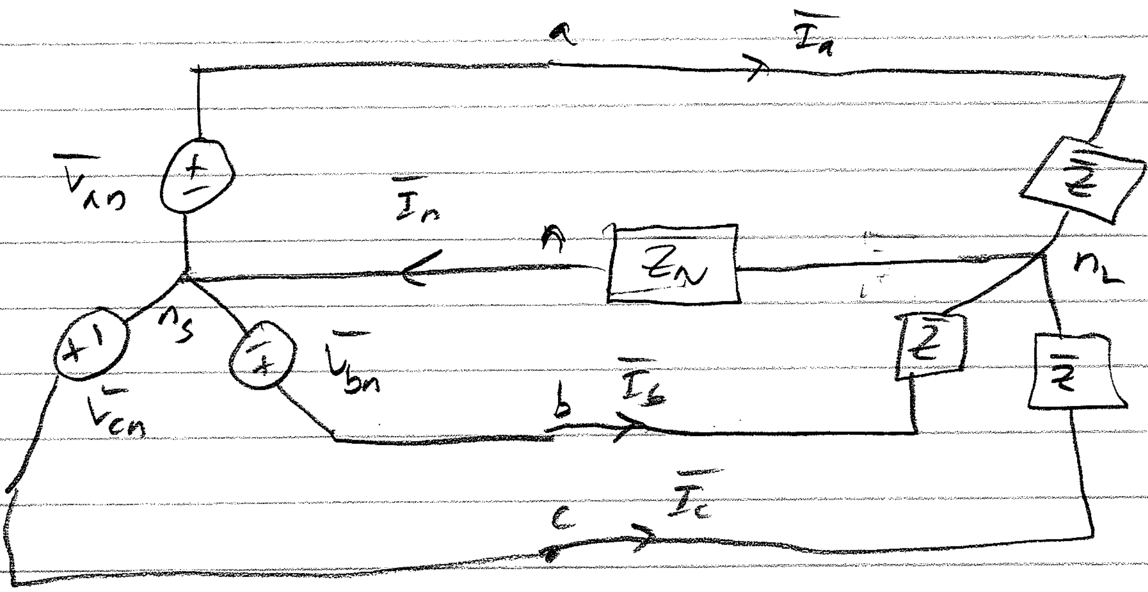


$$\boxed{V_L = \sqrt{3} V_\phi}$$

2006

Similarly for $\bar{V}_{bc}$ & $\bar{V}_{ca}$

4-wire wye source + 4 wire wye load (balanced)



note: $I_{Line} = I_{phase}$ (I_ϕ)

$$-\bar{V}_{an_S} + \bar{I}_a \bar{Z} + \bar{I}_n \bar{Z}_n = 0$$

$$-\bar{V}_{bn_S} + \bar{I}_b \bar{Z} + \bar{I}_n \bar{Z}_n = 0$$

$$-\bar{V}_{cn_S} + \bar{I}_c \bar{Z} + \bar{I}_n \bar{Z}_n = 0$$

add

$$0 + \bar{Z}(\bar{I}_a + \bar{I}_b + \bar{I}_c) + 3\bar{I}_n \bar{Z}_n = 0$$

but by KCL $\bar{I}_a + \bar{I}_b + \bar{I}_c = \bar{I}_n$

so: $\bar{I}_n(\bar{Z} + 3\bar{Z}_n) = 0$

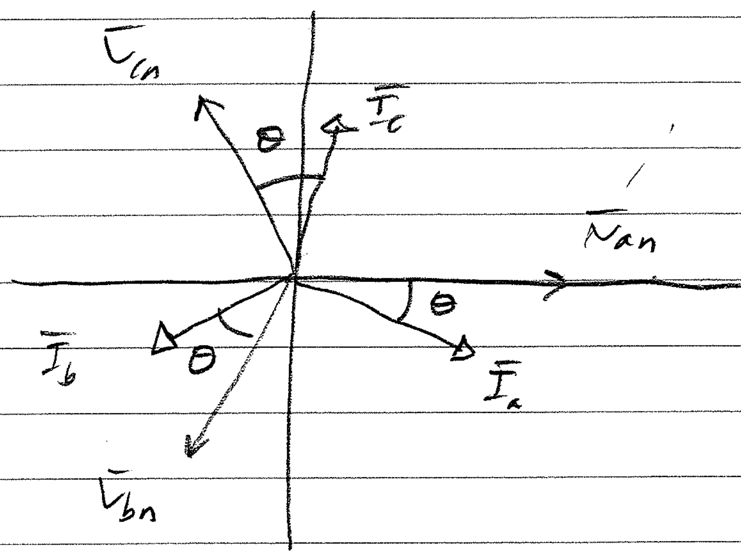
so: $\bar{I}_n = 0$ so $\bar{V}_{an_L} = \bar{V}_{an_S}$ etc.

$$\bar{I}_a = \frac{\bar{V}_{an_S}}{\bar{Z}} \text{ etc.}$$

#5

Let $\bar{Z} = Z\angle\theta$ and $\theta_v = 0$

$$\bar{I}_a = \frac{V_\phi\angle 0}{Z\angle\theta} = I_\phi\angle -\theta = I_L\angle -\theta$$



$$\bar{S}_{3\phi} = \bar{S}_a + \bar{S}_b + \bar{S}_c + \bar{S}_n$$

$$= \bar{V}_{anL}\bar{I}_a^* + \bar{V}_{bnL}\bar{I}_b^* + \bar{V}_{cnL}\bar{I}_c^* + \bar{V}_{nLnS}\xrightarrow{0}\bar{I}_n$$

$$= \bar{V}_{ans}\bar{I}_a^* + \bar{V}_{bns}\bar{I}_b^* + \bar{V}_{cns}(-\bar{I}_a - \bar{I}_b)$$

$$= \bar{V}_{ac}\bar{I}_a^* + \bar{V}_{bc}\bar{I}_b^* = \bar{V}_{bc}\bar{I}_b - \bar{V}_{ca}\bar{I}_a^*$$

$$= \sqrt{3}V_\phi\angle -90^\circ \; I_L\angle +170^\circ + \theta - \sqrt{3}V_\phi\angle 150^\circ \; I_L\angle +\theta$$

$$= \sqrt{3}V_\phi I_L\left(1\angle 30 + \theta - 1\angle 150 + \theta\right)$$

$$\bar{S}_{3\phi} = \sqrt{3}\, V_\phi I_L \Big(\cos(30+\theta) + j\sin(30+\theta)$$
$$- \cos(150+\theta) - j\sin(150+\theta) \Big)$$

$$= \sqrt{3}\, V_\phi I_L \Big(\tfrac{\sqrt{3}}{2}\cos\theta - \cancel{\tfrac{1}{2}\sin\theta} + \cancel{j\tfrac{1}{2}\cos\theta} + j\tfrac{\sqrt{3}}{2}\sin\theta$$
$$+ \tfrac{\sqrt{3}}{2}\cos\theta + \cancel{\tfrac{1}{2}\sin\theta} - \cancel{j\tfrac{1}{2}\cos\theta} + j\tfrac{\sqrt{3}}{2}\sin\theta \Big)$$

$$= \sqrt{3}\, V_\phi I_L \left(\sqrt{3}\cos\theta + j\sqrt{3}\sin\theta \right)$$

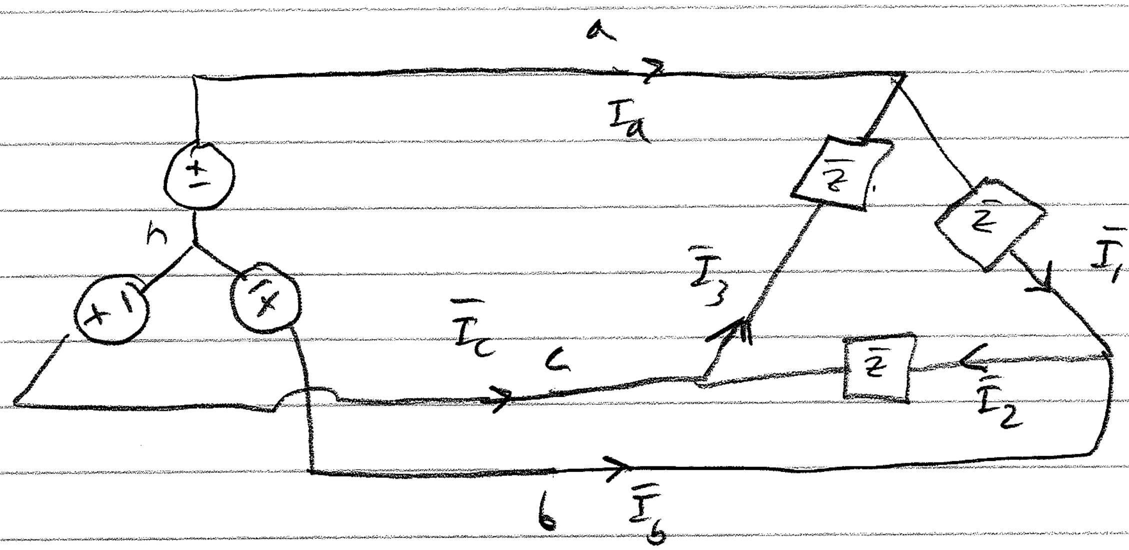
$$= 3\, V_\phi I_L \angle\theta$$

$$= \sqrt{3}\, V_L I_L \angle\theta$$

$$\boxed{\begin{aligned} P_{3\phi} &= \sqrt{3}\, V_L I_L \cos\theta \\ Q_{3\phi} &= \sqrt{3}\, V_L I_L \sin\theta \end{aligned}}$$

#6

3-wire Y_{source} supplying Δ load



$$\bar{I}_1 = \frac{\bar{V}_{ab}}{\bar{Z}} \qquad \bar{I}_2 = \frac{\bar{V}_{bc}}{\bar{Z}} \qquad \bar{I}_3 = \frac{\bar{V}_{ca}}{\bar{Z}}$$

Assume $\bar{V}_{ab} = V_L \angle 0$ (Same as assuming $\theta_V = -30^\circ$)
ABC Sequence

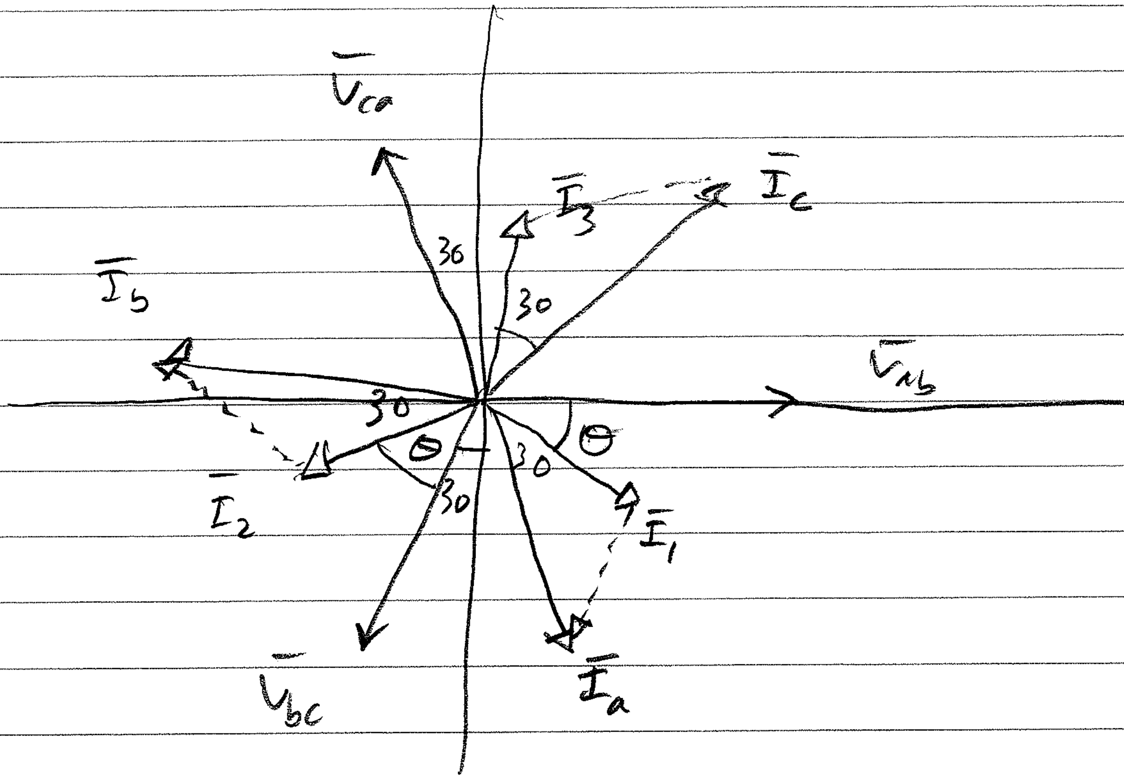
Assume $\bar{Z} = Z\angle\theta \qquad \theta = 45^\circ$

2003

$$\bar{I}_a = \bar{I}_1 - \bar{I}_3$$

$$\bar{I}_b = \bar{I}_2 - \bar{I}_1$$

$$\bar{I}_c = \bar{I}_3 - \bar{I}_2$$



For $\bar{I}_1 = I_\phi \angle -\theta$

$$\bar{I}_a = \sqrt{3}\, I_\phi \angle -\theta - 30^\circ = I_L \angle -\theta - 30^\circ$$

etc.

point is $\boxed{I_L = \sqrt{3}\, I_\phi}$

$$\bar{S}_{3\phi} = \bar{S}_{ab} + \bar{S}_{bc} + \bar{S}_{ca}$$

$$= \bar{V}_{ab}\,\bar{I}_1^{\,*} + \bar{V}_{bc}\,\bar{I}_2^{\,*} + \bar{V}_{ca}\,\bar{I}_3^{\,*}$$

$$= V_L\angle 0\; I_\phi\angle{+\theta} + V_L\angle{-120^\circ}\; I_\phi\angle{+120^\circ+\theta} + V_L\angle{+120^\circ}\; I_\phi\angle{-120^\circ+\theta}$$

$$= 3V_L\angle 0\; I_\phi\angle\theta$$

$$= \sqrt{3}\,V_L\,I_L\angle\theta$$

$$\boxed{\begin{aligned} P_{3\phi} &= \sqrt{3}\,V_L I_L \cos\theta \\ Q_{3\phi} &= \sqrt{3}\,V_L I_L \sin\theta \end{aligned}}$$

Sam as Y-Y

Example (Text problem 2.16)

$V_L = 345\,kV \qquad S_{3\phi} = 750\,mVA \qquad PF = 0.8 \text{ lag}$

Delta connected load

Find: a) $\bar{Z}_{\text{per phase}}$ b) I_L & I_ϕ c) P & Q per phase

d) $\bar{S}_{3\phi}$

Soln: Delta connected load, so $I_\phi = \dfrac{750\,mVA/3}{345\,kV}$

(a) $I_\phi = \dfrac{250 \times 10^6}{345 \times 10^3} = 725\,A$

$Z = \dfrac{345\,kV}{725\,A} = 476\,\Omega$

$\theta = +\cos^{-1} .8 = 37^\circ$

$$\boxed{\bar{Z}_{\text{per phase}} = 476\,\angle 37^\circ}$$

(b) $I_\phi = 725 \qquad I_L = \sqrt{3}\,I_\phi = 1256\,A$

(c) $P_{\text{per phase}} = 345k \times 725 \times .8 = 200\,mW$

$Q_{\text{per phase}} = 345k \times 725 \times \sin(37^\circ) = 150\,mVAR$

(d) $\bar{S}_{3\phi} = (3 \times 200 + j3 \times 150)$ MVA

$= 600 + j450$ MVA

$= 750 \angle 37^\circ$

Check: $\theta = 37^\circ$ ✓

Check: $P_{3\phi} = \sqrt{3} \times 345k \times 1256 \times \cos(37^\circ) \approx 600$ MW ✓

$Q_{3\phi} = \sqrt{3} \times 345k \times 1256 \sin(37^\circ) \approx 450$ MVAR ✓

$\bar{S}_{3\phi} = \sqrt{3} \times 345k \times 1256 \angle 37^\circ$

$= 750 \angle 37^\circ$ MVA ✓

Example (HW 2.22)

$$\bar{S}_{3\phi} = 15k \angle -\cos^{-1} 0.8 + \frac{72k}{0.8} \angle +\cos^{-1} 0.8$$

$$= 15k \angle -37^\circ + 90k \angle 37^\circ$$

$$= 12{,}000 - j9600 + 71{,}900 + j54{,}200$$

$$= 83{,}900 + j45{,}200 = 95{,}300 \angle 28^\circ$$

a) $95{,}300 = \sqrt{3} \times 2000 \times I_L$ $\qquad I_L = 27.5\,A$

$$PF = \cos 28^\circ = 0.88 \text{ lag}$$

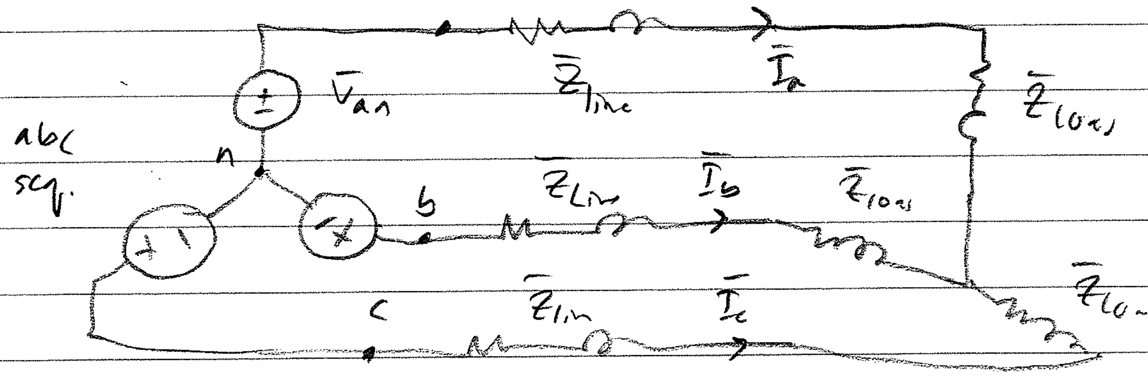
b) need 45.2 kVAR to bring PF to unity

#7

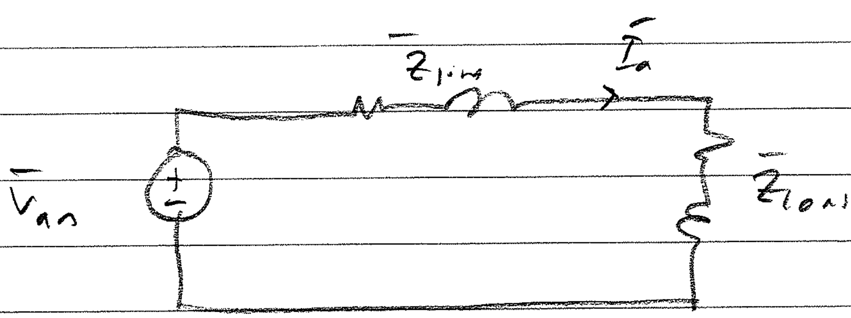
Per-phase equivalents

For Y-Y

Just use phase a quantities



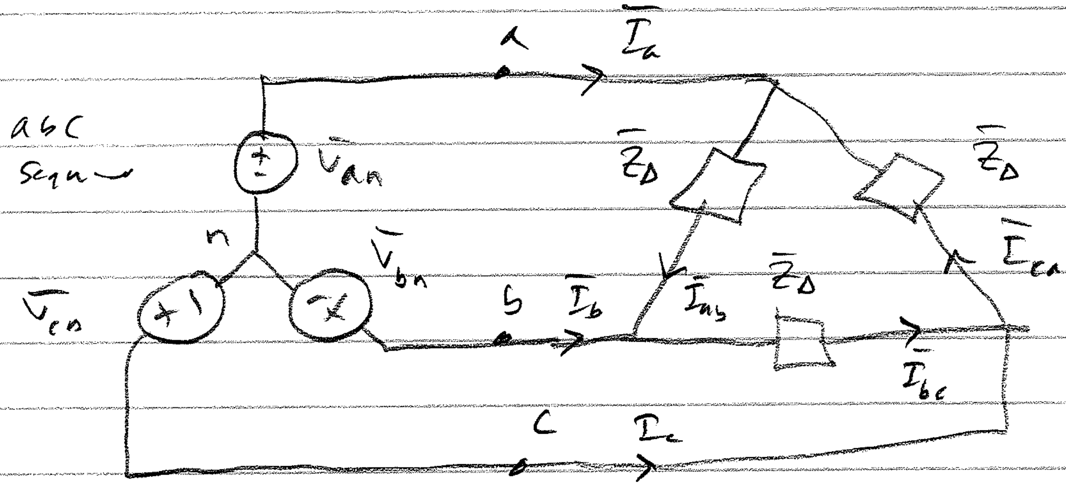
$\Downarrow$



$$\bar{I}_a = \frac{\bar{V}_{an}}{\bar{Z}_{line} + \bar{Z}_{load}}$$

Then rotate 120° for $\bar{I}_b$ $\bar{I}_c$

For Y Δ

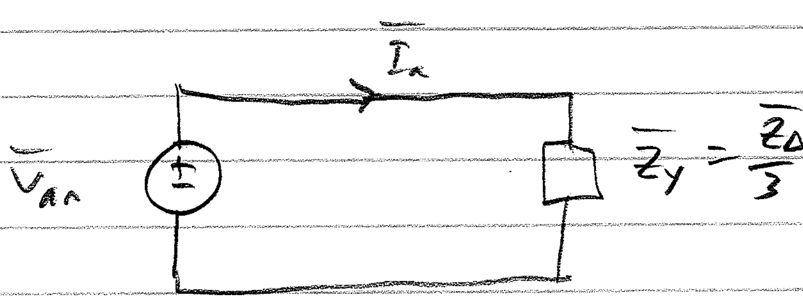


$$\bar{V}_{an} = V\angle 0 \rightarrow \bar{V}_{ab} = \sqrt{3}V\angle 30$$
$$\bar{V}_{bc} = \sqrt{3}V\angle -90$$
$$\bar{V}_{ca} = \sqrt{3}V\angle 150^\circ$$

$$\bar{I}_{ab} = \frac{\sqrt{3}V\angle 30}{\bar{Z}_\Delta} \qquad \bar{I}_{bc} = \frac{\sqrt{3}V\angle -90}{\bar{Z}_\Delta} \qquad \bar{I}_{ca} = \frac{\sqrt{3}V\angle 150}{\bar{Z}_\Delta}$$

$$\bar{I}_a = \bar{I}_{ab} - \bar{I}_{ca} = \sqrt{3}\angle -30^\circ\, \bar{I}_{ab} = \frac{3V\angle 0}{\bar{Z}_\Delta} = \frac{\bar{V}_{an}}{\bar{Z}_\Delta/3} = \frac{\bar{V}_{an}}{\bar{Z}_y}$$

$$\bar{Z}_y = \frac{\bar{Z}_\Delta}{3}$$



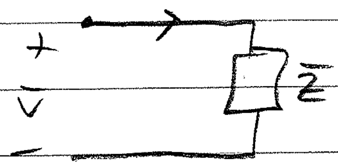
Example (text 2.23)

$$\bar{S}_{3\phi} = 1060k\angle 37^\circ = 800k + j600k \qquad V_L = 4160\,v$$

$$\bar{S}_{\Delta\,load} = (800k + j600k) - (200k + j0)$$

$$= 600k + j600k = 848k\angle 45^\circ$$

Note!



$$\bar{S} = \bar{V}\bar{I}^* = \bar{V}\left(\frac{\bar{V}}{\bar{Z}}\right)^* = \frac{|\bar{V}|^2}{Z^*}$$

$$\frac{600k + j600k}{3} = \frac{4160^2}{\bar{Z}^*} \qquad (1\phi\ \Delta\ \text{connected})$$

$$\bar{Z} = \frac{4160^2 \times 3}{600k - j600k} = \frac{51.9\times 10^6}{600\sqrt{2}k\angle -45^\circ}$$

$$= 61.2\angle 45^\circ\,\Omega = 43.3 + j43.3\,\Omega$$

Note: $1000k = \sqrt{3}\times 4160 \times I_{L\,TOTAL}$

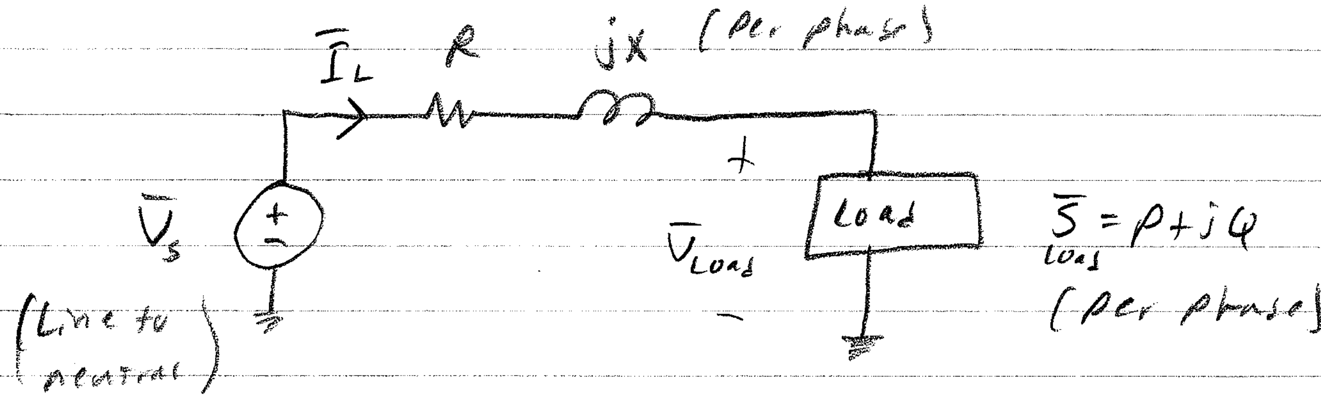
$I_{L\,TOTAL} = 139A$

$I_{L\,Y} = I_{\phi\,Y} = \frac{200k}{\sqrt{3}\times 4160} = 28A$

$I_{L\,\Delta} = \frac{848k}{\sqrt{3}\times 4160} = 118A \qquad I_{\phi\,\Delta} = \frac{118}{\sqrt{3}} = 68A \qquad \left(\text{Checks with } \frac{4160\,v}{61\Omega}\right)$

Suppose the "Line" has an impedance

Per phase circuit



Given: P, Q, R, X, V_{load} (magnitude)

Find: V_S (magnitude)

assume this

Soln: $\bar{S}_{Load} = P + jQ = V_{Load}\angle 0 \ \bar{I}_L^*$

$$\bar{I}_L = \left(\frac{\bar{S}_{Load}}{V_{Load}\angle 0} \right)^*$$

Then: $\bar{V}_S = (R + jX)\bar{I}_L + \bar{V}_{Load}$

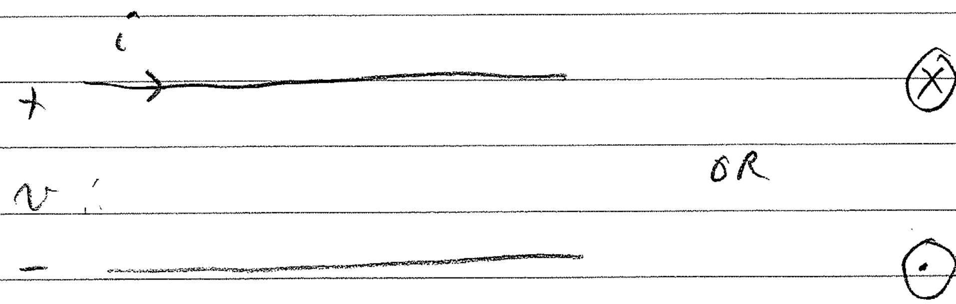
— Work other text problems

#8

Chapter 3

Magnetic Circuits

Review of EM



Normally taught that i produces H, B (Ampere)

v produces E, D (Faraday)

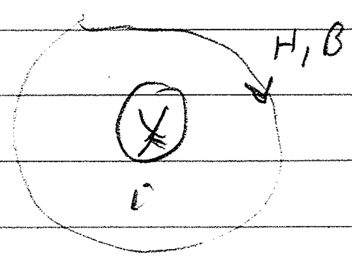
Example

could be 3 dimensional

Amperes Law $\oint_C \underline{H} \cdot \underline{dl} = \int_S \underline{J_f} \cdot \underline{n} \, da$

"C" encloses "S" $\underline{n} \perp S$ IAW RHR dir of C

Assume uniform H, B around conductor



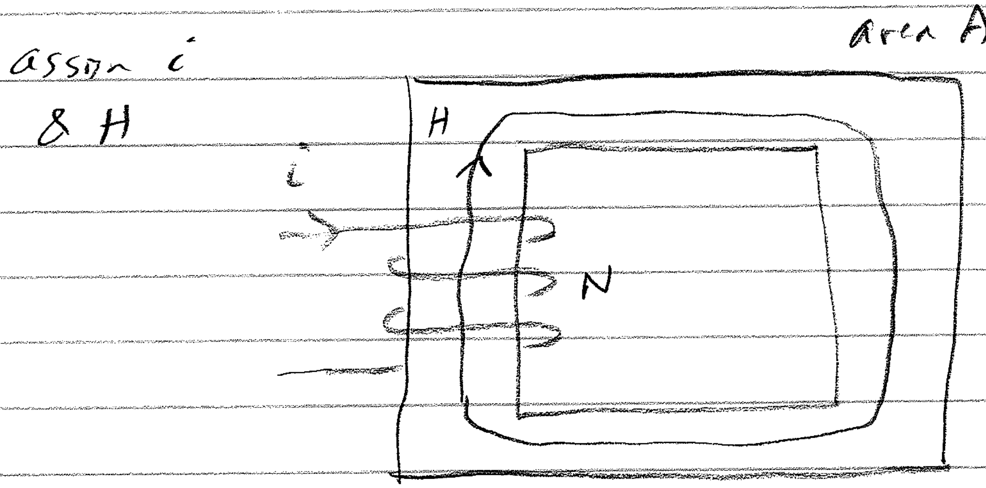
$Hl = i$

$H = \frac{i}{l}$

The B and H go in air to get around the conductor

$\boxed{\text{Ampere's Law}}$ $\oint_C \underline{H} \cdot \underline{dl} = \int_S \underline{J_f} \cdot \underline{n}\, da$ "C" encloses "S"

$\underline{n} \perp$ to S IAW RHR of dir of C



Assume most H is in iron because μ is large

Take contour around H path

$$Hl = Ni \quad (\text{mmf} - \mathcal{F})$$

$$H = \frac{Ni}{l} \quad \text{Amp "Turns"/m}$$

stop 2005 + 2012

Assume linear $B = \mu H = \frac{\mu Ni}{l}$ webers/m² (Tesla)

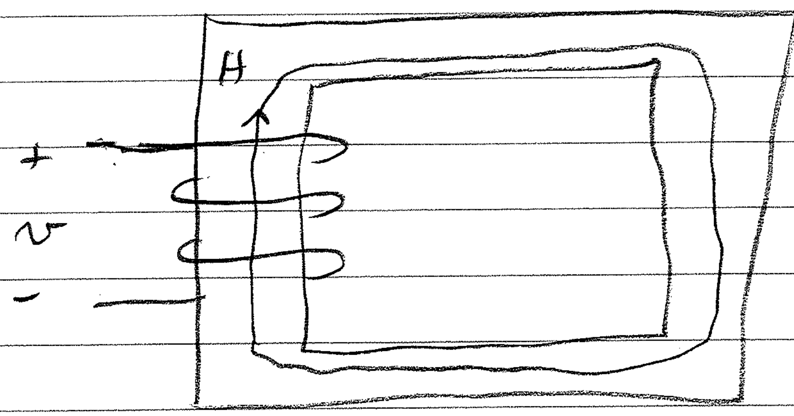
$$\phi = BA = \underbrace{\frac{\mu A}{l}}_{P} Ni \quad \text{wb}$$

P (permeance) (Reluctance) $R = \frac{1}{P}$

$\boxed{\text{Faraday's Law}}$ $\quad \oint_C \underline{E} \cdot \underline{dl} = -\frac{d}{dt} \underbrace{\int_S \underline{B} \cdot \underline{n}\, da}_{\lambda}$

$\underline{n} \perp$ to S IAW RHR DIR of C

assign v
& H



Assume
$\sigma \to \infty$
(no R)

$J = \sigma E$

J finite $\quad E_{wire} \to 0$

$-v + 0 = -\frac{d}{dt} \underbrace{NBA}_{\lambda}$

$$v = N \frac{d\phi}{dt} = \frac{d\lambda}{dt} \qquad (\lambda = N\phi)$$

Put these two together:

$$v = N^2 \frac{\mu A}{l} \frac{di}{dt} = L \frac{di}{dt}$$

#9

Example

$v = 120\sqrt{2}\cos 2\pi 60t$

$N = 100$ turns

$A = .0049\ m^2$

$l = 40\ cm$

$\mu = 1000\mu_0 = 1000 \times 4\pi \times 10^{-7}\ H/m$

First: i (steady state)

$120\sqrt{2}\cos(2\pi 60t) = 100\dfrac{d\phi}{dt}$

$\phi = \dfrac{120\sqrt{2}\sin 2\pi 60t}{100 \times 2\pi 60} = .0045\sin 2\pi 60t\ wb$ (steady state)

$B = \dfrac{\phi}{A} = \dfrac{.0045\sin 2\pi 60t}{.0049} = 0.92\sin 2\pi 60t\ \ T$

$H = \dfrac{B}{\mu} = \dfrac{0.92\sin 2\pi 60t}{1000 \times 4\pi \times 10^{-7}} = 731\sin 2\pi 60t\ \ \dfrac{\text{Amp "turns"}}{m}$

$P = \dfrac{\mu A}{l} = \dfrac{1000 \times 4\pi \times 10^{-7} \times (.07)^2}{.4} = 1.54 \times 10^{-5}\ \ H$

$R = \dfrac{1}{P} = 64,961\ \ H^{-1}$

$$L = N^2 \mathcal{P} = 100^2 \times 1.54 \times 10^{-5} = 0.154 \text{ H}$$

$$v = L \frac{di}{dt}$$

$$120\sqrt{2} \cos 2\pi 60t = 0.154 \frac{di}{dt}$$

$$i = \frac{120\sqrt{2} \sin 2\pi 60 t}{0.154 \times 2\pi 60} = 2.9 \sin 2\pi 60 t$$

(steady state)

$$\lambda = Li = 0.154 \times 2.9 \sin 2\pi 60 t$$

$$= 0.45 \sin 2\pi 60 t \text{ wb "turns"}$$

Check $v = \frac{d\lambda}{dt} = 170 \cos 2\pi 60 t$ ✓

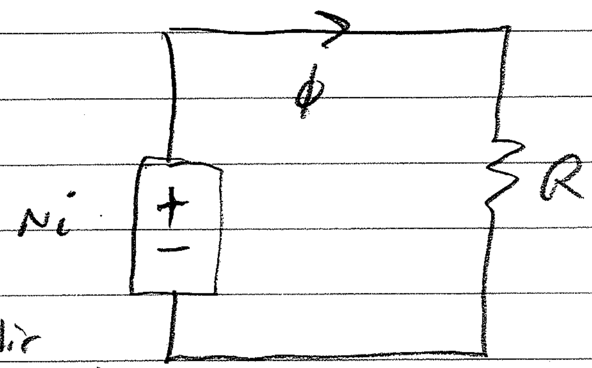
#10

Equivalent Magnetic Circuit

mmf ($\mathcal{F}$) $\Rightarrow$ Voltage (v)

flux (ϕ) $\Rightarrow$ current (i)

Reluctance ($\mathcal{R}$) $\Rightarrow$ resistance (R)



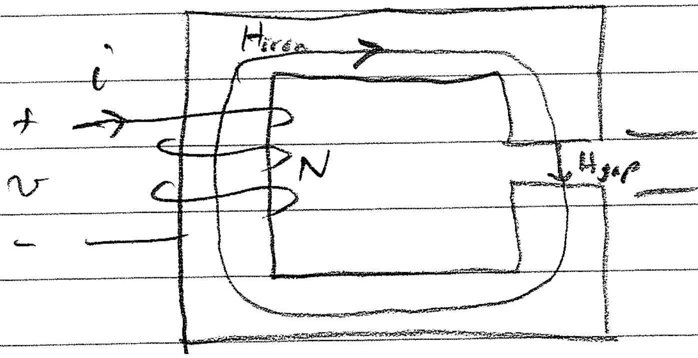
Thumb in dir of i (in coil)
RHR fingers go − to +

$$Ni = \mathcal{R}\phi$$

For $i = 2.9\sin 2\pi 60t$, $N = 100$, $\mathcal{R} = 64{,}961$

$$\phi = \frac{100 \times 2.9\sin 2\pi 60t}{64{,}961} = 0.00455\sin 2\pi 60t \checkmark$$

Add an air gap to iron



Assume all flux takes full path

Conservation of flux

One B; two H

Area A

Ampere's law

$$\oint_C \underline{H} \cdot \underline{dl} = \int_S \underline{J} \cdot \underline{n}\, da$$

S and C and $\underline{n} \perp$ to S & RHR dir of C

$$H_{iron} l_{iron} + H_{gap} g = Ni \qquad (\text{mean path } l)$$

Conservation of flux $\oint_S \underline{B} \cdot \underline{n}\, da = 0$

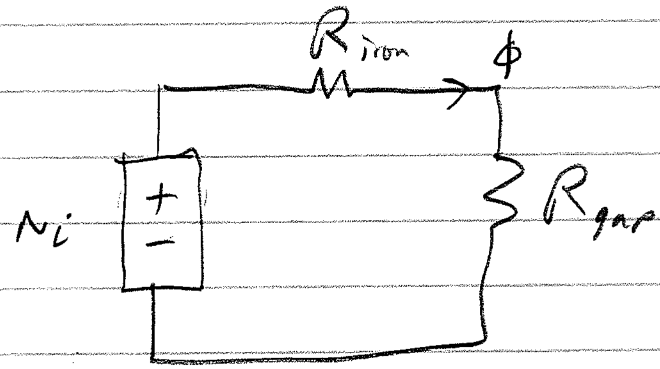
$\underline{n} \perp$ to S OUT

$$\phi = B_{iron} A_{iron} = B_{gap} A_{gap}$$

$$B_{iron} = \mu_{iron} H_{iron} \qquad B_{gap} = \mu_0 H_{gap}$$

$$H_{iron} = \frac{\phi}{\mu_{iron} A_{iron}} \qquad H_{gap} = \frac{\phi}{\mu_0 A_{gap}}$$

$$\left(\frac{l_{iron}}{\mu_{iron} A_{iron}}\right)\phi + \left(\frac{g}{\mu_0 A_{gap}}\right)\phi = Ni$$



Note

$\mathcal{R}_{iron} \ll \mathcal{R}_{gap}$

$$\phi = \frac{Ni}{\mathcal{R}_{iron} + \mathcal{R}_{gap}}$$

Faraday's Law

$$\oint_C \underline{E}\cdot d\underline{l} = -\frac{d}{dt}\int_S \underline{B}\cdot\underline{n}\,da$$

S and C, $\underline{n}$ ⊥ to S IAW RHR DIR OF C

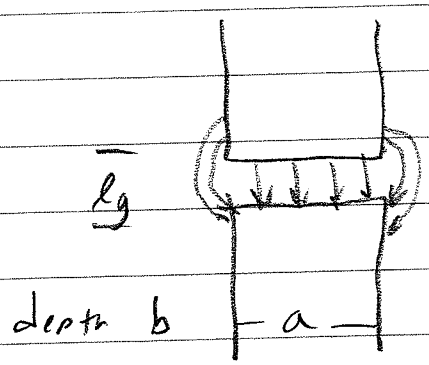
$$-v + 0 = -\frac{d}{dt}\lambda = -\frac{d}{dt}N\phi$$

$$v = N\frac{d\phi}{dt} = \underbrace{\frac{N^2}{\mathcal{R}_{iron} + \mathcal{R}_{gap}}}_{L}\frac{di}{dt}$$

Let $\mu_{iron} \to \infty$

$$L \approx \frac{N^2 \mu_0 A}{g} = N^2 P_{gap}$$

Fringing



flux forces to "outside" of gap makes the area bigger

$$A_c = ab$$

$$A_{gap} = (a + \ell_g)(b + \ell_g)$$

(empirical)

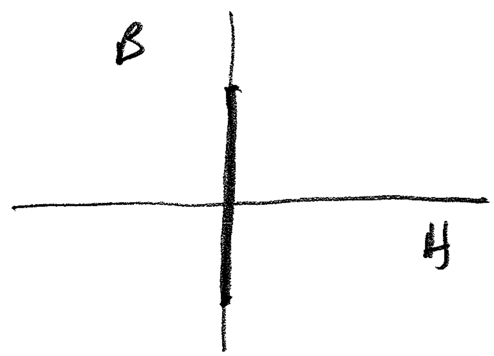
Explain what it means to have:

$$\mu \to \infty \quad (H = 0)$$

— Do parallel paths

Note: If $\mu \to \infty$, $H \to 0$

i.e.

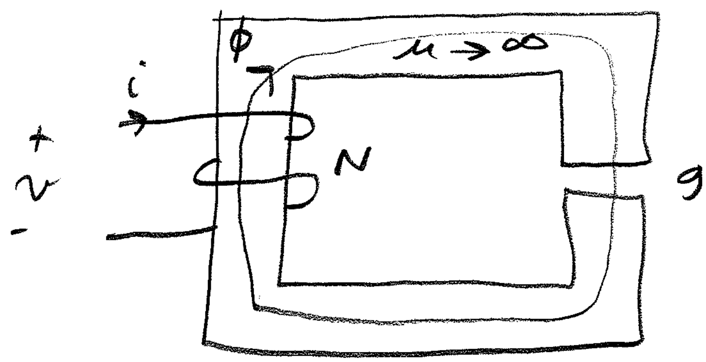


B is finite and

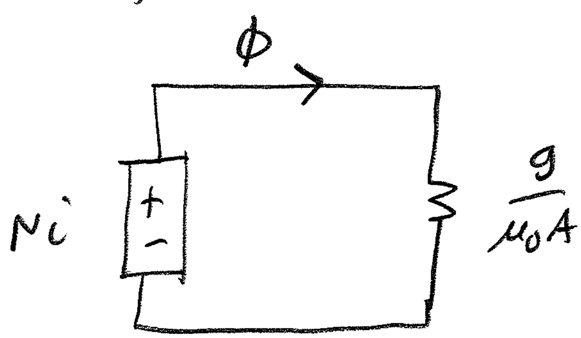
H is zero

$\mathcal{R}$ is zero

Example



$$v = N \frac{d\phi}{dt} \qquad \phi = B_{iron} A = B_{gap} A$$

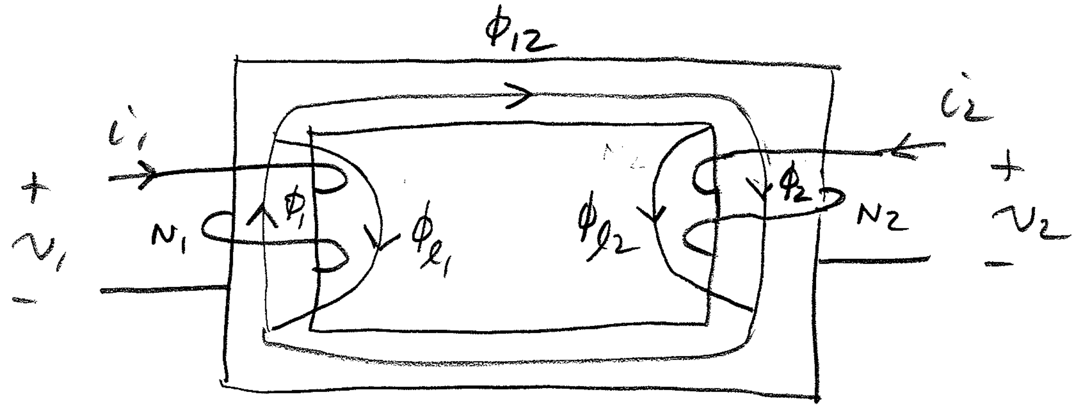


$$\phi = \frac{Ni}{(g/\mu_0 A)} = \frac{\mu_0 A N}{g} i$$

$$\lambda = \frac{\mu_0 A N^2}{g} i$$

mutual inductance

(coupled coils)



Consider 3 flux paths ⇒ 5 fluxes

ϕ_1 = total flux linking coil #1

ϕ_{12} = flux which links both coil #1 & coil #2

ϕ_{l_1} = flux which links coil #1 only

ϕ_{l_2} = flux which links coil #2 only

Faraday's Law:

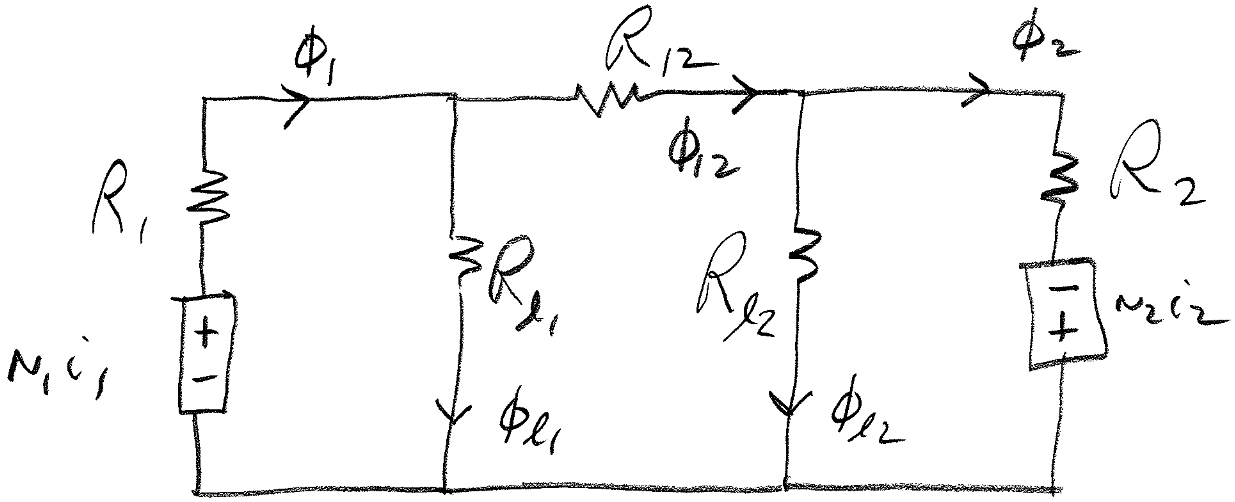
$$v_1 = \frac{d\lambda_1}{dt} \qquad v_2 = \frac{d\lambda_2}{dt}$$

$$\lambda_1 = N_1\phi_1 \qquad \lambda_2 = N_2\phi_2$$

depends on assumed voltage polarities and flux assumed directions

(now find ϕ_1 and ϕ_2)

Magnetic Equivalent Circuit



$$-N_1 i_1 + R_1\Phi_1 + R_{\ell_1}\Phi_{\ell_1} = 0$$

$$R_{12}\Phi_{12} + R_{\ell_2}\Phi_{\ell_2} - R_{\ell_1}\Phi_{\ell_1} = 0$$

$$R_2\Phi_2 - N_2 i_2 - R_{\ell_2}\Phi_{\ell_2} = 0$$

$$\Phi_1 - \Phi_{12} - \Phi_{\ell_1} = 0$$

$$\Phi_{12} - \Phi_2 - \Phi_{\ell_2} = 0$$

Each $R = \dfrac{\ell}{\mu A}$

(not so obvious for leakage paths)

5 Equations

5 unknowns

Solve these 5 linear equations for the 5 fluxes will get:

$$\phi_1 = a N_1 i_1 + b N_2 i_2$$

$$\phi_2 = b N_1 i_1 + c N_2 i_2$$

Then:

$$\lambda_1 = N_1 \phi_1 = a N_1^2 i_1 + b N_1 N_2 i_2$$

$$\lambda_2 = N_2 \phi_2 = b N_1 N_2 i_1 + c N_2^2 i_2$$

OR

Self inductance coil #1

mutual inductance 1 to 2

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2$$

mutual inductance 2 to 1

Self inductance coil #2

In this case, $|L_{12}| = |L_{21}| = M$ (a positive number)

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$\times$ don't show this!

$$k \triangleq \frac{M}{\sqrt{L_1 L_2}}$$

$$0 \le k \le 1$$

$k = 0$ no coupling (all leakage) $\rightarrow$ $M = 0$

$k = 1$ full coupling $\rightarrow$ $M = \sqrt{L_1 L_2}$ (no leakage)

Note: M is never negative

There may be cases where $L_{12} = -M$, or $L_{21} = -M$, but M is always ≥ 0

For example:

In the last derivation, suppose we change the sign on i_2 using $i_2' = -i_2$

The equations then are:

$$\boxed{\begin{aligned} v_1 &= L_1 \frac{di_1}{dt} - M \frac{di_2'}{dt} \\ v_2 &= M \frac{di_1}{dt} - L_2 \frac{di_2'}{dt} \end{aligned}}$$

and, if we reverse the polarity on v_2 using $v_2' = -v_2$

$$\boxed{\begin{aligned} v_1 &= L_1 \frac{di_1}{dt} - M \frac{di_2'}{dt} \\ v_2' &= -M \frac{di_1}{dt} + L_2 \frac{di_2'}{dt} \end{aligned}}$$