LECTURE 7

PER-PHASE CIRCUITS AND INTRODUCTION TO MAGNETICS (1)

Acknowledgment-
These handouts and lecture notes given in class are based on material from Prof. Peter Sauer’s ECE 330 lecture notes. Some slides are taken from Ali Bazi’s presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.
PER-PHASE EQUIVALENTS
PER-PHASE EQUIVALENTS

In balanced three phase circuits, it is preferable to work with per-phase equivalents and then convert the variables to three-phase quantities.

\[ I_a = \frac{V_{an}}{Z_{Line} + Z_{Load}} \]

\[ V_{an} \]
PER-PHASE EQUIVALENTS

Δ-Y CONVERSION

• Per-Phase equivalent circuits are very convenient for analyzing three-phase circuits.
• For a Y-source the load could be either Y or Δ.
• The load seen between two phases, e.g., a and b, can be expressed as:

\[
\bar{Z}_\Delta / 2\bar{Z}_\Delta = 2\bar{Z}_Y
\]

\[\Rightarrow \bar{Z}_\Delta = 3\bar{Z}_Y\]
PER-PHASE EQUIVALENTS

• The per-phase circuits can then be shown as follows:

\[ S_{\Delta,1\varphi} = \bar{V}_{ab} \bar{I}_a^* = \frac{V_{ab}^2}{\bar{Z}_\Delta} \]

\[ S_{Y,1\varphi} = \bar{V}_{an} \bar{I}_a^* = \frac{V_{an}^2}{\bar{Z}_Y} \]

\[ S_{Y,1\varphi} = S_{\Delta,1\varphi} \]
PER-PHASE EQUIVALENTS

• For either load, the voltage and load can be transformed to a Y-per-phase circuit, or Δ-per-phase circuit, and the $S$ should be the same.

• Example: $\bar{Z}_\Delta = 10 + j5$ Ω, $|V_{ab}| = 208$ V

\[
\begin{align*}
S_{Y,1\phi} &= \frac{V_{an}^2}{Z_Y} = \frac{\left(\frac{V_{ab}}{\sqrt{3}}\right)^2}{\bar{Z}_\Delta} = \frac{V_{ab}^2}{3\bar{Z}_\Delta} = 3.87 \angle 26.6^\circ \text{kVA}
\end{align*}
\]
EXAMPLE 2.17

The following two three-phase loads are connected in parallel across a three-phase 480 V wye-connected supply.
EXAMPLE 2.17

- **Load 1**: 24 kW at 0.8 PF lag (wye-connected)
- **Load 2**: 30 kVA at 0.8 PF lead (delta-connected)

Find the line currents $\bar{I}_{L1}$ and $\bar{I}_{L2}$ for each of the two loads, total complex power $\bar{S}_T$ and total line current.

Take $\bar{V}_{an}$ as reference.

Triangle method:
EXAMPLE 2.17

$$\bar{S}_{T_1} = \frac{24 \times 10^3}{0.8} (0.8 + j 0.6) \text{VA} = 24,000 + j18,000 \text{ VA}$$

$$\bar{S}_{T_2} = 30 \times 10^3 (0.8 - j 0.6) = 24,000 - j18,000 \text{ VA}$$

$$\bar{S}_T = \bar{S}_{T_1} + \bar{S}_{T_2} = 48,000 \text{ W}$$
EXAMPLE 2.17

Line current:

\[(\sqrt{3})V_L I_{L1} \cos \theta = 24,000\]

\[I_{L1} = \frac{24,000}{(480)(\sqrt{3})(0.8)} = 36.08 \ \text{A}\]

\[I_{L1} = 36.08 \angle -36.78^\circ \text{ since the current is lagging}\]

\[(\sqrt{3})V_L I_{L2} = 30,000 \Rightarrow I_{L2} = \frac{30,000}{(\sqrt{3})(480)} = 36.08 \ \text{A}\]
EXAMPLE 2.17

\[ I_{L2} = 36.08 \angle 36.78^\circ \text{ since the current is leading} \]

\[ Q_T = Q_{T1} + Q_{T2} = 18000 - 18000 = 0 \]

\[ (\sqrt{3})V_L I_L = 48,000 \Rightarrow I_L = \frac{48,000}{(480)(\sqrt{3})} = 57.7 \text{ A} \]

Per-phase equivalent method

\[ \frac{480}{\sqrt{3}} \angle 0^\circ = 277.13 \angle 0^\circ \text{V} \]

\[ 8 \text{ kW at 0.8 PF lag} \]

\[ 10 \text{ kVA at 0.8 PF lead} \]
EXAMPLE 2.17

Phase-to-neutral voltage is $480/\sqrt{3} = 277.13$ V

$$8 \times 10^3 = (277.13)I_{L1}(0.8) \Rightarrow I_{L1} = 36.08 \text{ A}$$

$I_{L1} = 36.08 \angle -36.78^\circ$ since the PF is lag

$$(10 \times 10^3)(0.8) = (277.13)(I_{L2})(0.8) \Rightarrow I_{L2} = 36.08 \text{ A}$$

$\bar{I}_{L2} = 36.08 \angle 36.78^\circ$ A since the PF is leading

$\bar{I}_L = \bar{I}_{L1} + \bar{I}_{L2} = 36.08 \angle -36.78^\circ + 36.08 \angle 36.78^\circ = 57.7 \angle 0^\circ$ A
MAGNETIC CIRCUITS

• Maxwell’s Equations:
  – Ampere’s Law: \( \oint_{C} H \, dl = \int_{S} J \cdot n \, da \)
  The magnetic field in any closed circuit is proportional to the electric current flowing through the loop.
  – Faraday’s Law: \( \oint_{C} E \, dl = -\int_{S} \frac{dB}{dt} \cdot n \, da \)
  The line integral of the electric field around a closed loop is equal to the negative of the rate of change of the magnetic flux through the area enclosed by the loop.

9/14/2017
ECE ILLINOIS
Copyright © 2017 Hassan Sowidan
MAGNETIC CIRCUITS

– Conservation of Charge:  \[ \oint J \cdot n \, da = 0 \]

– Gauss’s Law:  \[ \oint \mathbf{B} \cdot n \, da = 0 \]

The net magnetic flux out of any closed surface is zero.

(for a magnetic dipole, in any closed surface the magnetic flux inward toward the south pole will equal the flux outward from the north pole).

Source: study.com
MAGNETIC CIRCUITS

• What do these symbols mean?
  – Integral over a closed contour $C$: $\oint_C$
  – Surface $S$ define by $C$: $\int_S$
  – Integral over a closed surface $S$: $\oint_S$
  – Length of the contour $C$: $\int_C dl$
  – $H$ is the magnetic field intensity (A.turns/m)
  – $B$ is the magnetic flux density (Tesla or Wb/m$^2$)
  – $E$ is the electric field (V/m)
  – $J$ is the current density (A/m$^2$)
  – $n$ is the normal vector to $S$. 
READING MATERIAL

• Reading material: Sections 2.6, 3.1, and 3.2.

• Recommended reading for next time: Continue Section 3.2, and Section 3.3.