LECTURE 2
ACTIVE, REACTIVE, AND COMPLEX POWER

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer’s ECE 330 lecture notes. Some slides are taken from Ali Bazi’s presentations

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.
TWO-TERMINAL NETWORK

• A two-terminal electrical network has voltage at its terminals and current flowing in and out of its terminals.

\[
p(t) = v(t)i(t)
\]

• The instantaneous power is \( p(t) = v(t)i(t) \).

• For \( i(t) = I_m \cos(\omega t + \theta_i) \) A and \( v(t) = V_m \cos(\omega t + \theta_v) \) V we get

\[
p(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)
\]
TWO-TERMINAL NETWORK

\[ \cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)] \]

\[ p(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) \quad \text{W} \]

The first term is time-independent, while the second term is a sinusoid at double frequency.
TWO-TERMINAL NETWORK

• The average power is thus

\[ P = \frac{1}{T} \int_0^T P(t) \, dt \quad , \quad T = \frac{2\pi}{\omega} \]

\[ P_{in} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i). \]

• This is called the active or real power and its unit is watts (W).

• The power factor is the cosine of the phase angle between \( v(t) \) and \( i(t) \).
POWER FACTOR

• The power factor \((P.F.)\) is thus \(P.F. = \cos(\theta_v - \theta_i)\).

• The power factor can be:
  
  – Lagging: \(0^\circ < \theta_v - \theta_i < 90^\circ\)
  
  – Leading: \(-90^\circ < \theta_v - \theta_i < 0^\circ\)
  
  – Unity: \(\theta_v - \theta_i = 0\)

Therefore, \(0 \leq P.F. \leq 1\), and the highest real power exists when \(P.F. = 1\).
APPARENT POWER AND REACTIVE POWER

• The apparent power is \( S = \frac{V_m I_m}{2} \)

• The apparent power unit is volt-amps (VA).

• The reactive power is \( Q_{in} = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \).

• The reactive power unit is volt-amps-reactive (VARs).
COMPLEX POWER

• The instantaneous power is

\[ p(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) \text{ W} \]

• The time varying component

\[ \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) = \frac{V_m I_m}{2} \left\{ \cos[(2\omega t + 2\theta_i) + (\theta_v - \theta_i)] \right\} \]
\[ = \frac{V_m I_m}{2} \cos(2\omega t + 2\theta_i) \cos(\theta_v - \theta_i) - \frac{V_m I_m}{2} \sin(2\omega t + 2\theta_i) \sin(\theta_v - \theta_i) \]
COMPLEX POWER

• Define

\[ Q_{in} = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i), \quad \text{(Reactive power)} \]

\[ p(t) = P_{in} + P_{in} \cos(2\omega t + 2\theta_i) - Q_{in} \sin(2\omega t + \theta_i) \]

\[ = P_{in} (1 + \cos(2\omega t + \theta_i)) - Q_{in} \sin(2\omega t + 2\theta_i) \]

• The real power can be written as

\[ P_{in} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \]
PHASOR REPRESENTATION

\[ P_{in} = \text{Re}\left\{ \frac{V_m I_m}{2} e^{j\theta_v} e^{-j\theta_i} \right\} = \text{Re}\{V_{rms} e^{j\theta_v} I_{rms} e^{-j\theta_i}\} \]

- The reactive power can be written as

\[ Q_{in} = \text{Im}\left\{ \frac{V_m I_m}{2} e^{j\theta_v} e^{-j\theta_i} \right\} = \text{Im}\{V_{rms} e^{j\theta_v} I_{rms} e^{-j\theta_i}\} \]

- The voltages and currents can be written as phasors:

\[ V_{rms} e^{j\theta_v} = V \quad \text{and} \quad I_{rms} e^{j\theta_i} = I. \]

\[ P_{in} = \text{Re}(V^{\ast} I^{\ast}) = V_{rms} I_{ms} \cos(\theta_v - \theta_i) \]

\[ Q_{in} = \text{Im}(V^{\ast} I^{\ast}) = V_{rms} I_{ms} \sin(\theta_v - \theta_i) \]

Source: Tonex.com
Complex Power

• Define the complex power as \( \bar{S} = P_{in} + jQ_{in} \)

• Then \( \bar{S} \) can be written as \( \bar{S} = \bar{V} \bar{I}^* \)

• The quantity \( \bar{I}^* \) is the complex conjugate of \( \bar{I} \).

• \( \bar{S} \) can also be written as

\[
\bar{S} = S \angle (\theta_v - \theta_i)
\]

• Note that

\[
S = \frac{V_m I_m}{2} = \sqrt{P_{in}^2 + Q_{in}^2}
\]
ALTERNATE FORMS OF COMPLEX POWER

• If the load is $\bar{Z} = R + jX$, connected across the source $\bar{V}$

  By Ohm’s law: $\bar{V} = \bar{Z} \bar{I}$, but $\bar{S} = \bar{V} \bar{I}^*$

  Then $\bar{S}$ can be written as $\bar{S} = I^2 R + jI^2 X$ Also,

  $P = I^2 R$ and $Q = I^2 X$, $\bar{Z}$ and $P.F. = \cos(\text{angle}(\bar{Z}))$.

• Thus, $Q > 0$ when $\bar{Z}$ is inductive, $X = \omega L$

  and $Q < 0$ when $\bar{Z}$ is capacitive, $X = -\frac{1}{\omega C}$

• $\bar{S}$ and $\bar{Z}$ are not phasors but complex quantities.
EXAMPLE: LC FILTER AND R LOAD

• The circuit shown is commonly used as an LC filter to supply a load, which is resistive in this case.

• Find the current, real, reactive, and complex powers, and the P.F. for \( v(t) = \sqrt{2}V_{rms} \cos(377t) \)

\[
\bar{Z} = j\omega L + \left( R / / \frac{-j}{\omega C} \right)
\]

\[
\bar{Z} = \frac{\omega L + j(\omega^2 R L C - R)}{\omega R C - j}
\]
EXAMPLE: LC FILTER AND R LOAD

• Let

\[ V_{rms} = 120 \text{V}, \ L = 1 \text{mH}, \ C = 6.8 \text{mF}, \ \text{and} \ R = 10 \Omega. \]

\[ \bar{Z} = 0.0197 \angle -39.41^\circ = 0.0152 - j0.0125 \Omega \]

\[ \bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{120 \angle 0^\circ}{0.0197 \angle -39.41^\circ} = 6091.4 \angle 39.41^\circ \text{A} \]

\[ i(t) = 6091.4 \sqrt{2} \cos(377t + 39.41^\circ) \]

\[ \bar{S} = \bar{V} \ \bar{I}^* = 731 \angle -39.41^\circ \text{kVA} \]

\[ P_{in} = 731 \cos(-39.41^\circ) = 564.8 \text{kW} \]

\[ Q_{in} = 731 \sin(-39.41^\circ) = -464.1 \text{kVAR} \]

\[ \text{P.F.} = \cos(-39.41^\circ) = 0.773 \text{ leading (} \theta_v - \theta_i = -39.41^\circ \) \]
READING MATERIAL

• Reading material: Chapter 2 sections 2.1 – 2.3.

• Recommended reading for next time: section 2.4.