LECTURE 18
FORCES OF ELECTRIC ORIGIN – ENERGY APPROACH(2)

Acknowledgment- These handouts and lecture notes given in class are based on material from Prof. Peter Sauer’s ECE 330 lecture notes. Some slides are taken from Ali Bazi’s presentations.

Disclaimer- These handouts only provide highlights and should not be used to replace the course textbook.
ENERGY AND CO-ENERGY

the force of electric origin $f^e$ derived from either energy or co-energy expressions is identical. Depending on the system description, one or the other may be used.

In electrically linear system the energy and co-energy are numerically equal.
Suppose \( \lambda(i, x) = L(x) i \) which is electrically linear, then

\[
i = \frac{\lambda(i, x)}{L(x)}
\]

\[
W_m(\lambda, x) = \int_0^\lambda i(\hat{\lambda}, x) \left| \frac{d\hat{\lambda}}{x=\text{const}} \right.
\]

\[
W'_m(i, x) = \int_0^i \lambda(i, x) \left| \frac{di}{x=\text{const}} \right.
\]
ENERGY AND CO-ENERGY

Since \( \lambda(i, x) = L(x)i \), \( W_m \) and \( W_m' \) are equivalent.

For a given value of \( x \), the relationship between \( \lambda \) and \( i \) is linear.

\[
\text{Area } A = \int_o^\lambda i(\lambda, x) d\lambda \text{ is the } \text{"energy"}
\]

\[
\text{Area } B = \int_o^i \lambda(i, x) di \text{ is the } \text{"co-energy"}
\]

for linear system \( W_m = W_m' \).

And note

\[
\lambda i = W_m + W_m'
\]
ENERGY AND CO-ENERGY

If $\lambda(i, x)$ is a nonlinear function as shown, then area $A \neq B$ and $W_m \neq W'_m$ but still $\lambda i = W_m + W'_m$. 
MULTI-PORT SYSTEMS

For the case of multi-port system, we have the flux linkages as

\[ \lambda_1 = L_{11}(x)i_1 + L_{12}(x)i_2 \]
\[ \lambda_2 = L_{21}(x)i_1 + L_{22}(x)i_2 \]

Path from \( 0 \to i_1 \), \( 0 \to i_2 \), \( x_0 \to x \)

Must be path independent \( (L_{12} = L_{21} \quad for \ linear\ system) \)

Choose path x first while \( i_1 \) and \( i_2 \) are zero

\( i_1 \) next while \( x = \) constant , \( i_2 = 0 \)

\( i_2 \) next while \( x = \) constant , \( i_1 = i_1 \)
MULTI-PORT SYSTEMS

Computation of $W'_m$

$$W'_m(i_1, i_2, x) = \int_0^{i_1} \lambda_1 \, di_1 + \int_0^{i_2} \lambda_2 \, di_2 + \int_0^x f^e \, dx$$

with

$$L_{11}(x)i_1^2 + L_{21}(x)i_1i_2 + L_{22}(x)i_2^2$$

$$f^e(i_1, i_2, x) = \frac{\partial W'_m(i_1, i_2, x)}{\partial x}$$
EXAMPLE

A certain multiple-port rotational system has the following flux-current relations, where $\theta$ is a rotational variable

\[
\lambda_s = L_s i_s + M \cos \theta i_r \\
\lambda_r = M \cos \theta i_s + L_r i_r
\]

Compute the co-energy $W'_m$ and the torque of electrical origin

\[
W'_m(i_1, i_2, x) = \frac{1}{2} L_s i_s^2 + M \cos \theta i_r + \frac{1}{2} L_r i_r^2
\]

\[
T^e (i_s, i_r, \theta) = \frac{\partial W'_m (i_s, i_r, \theta)}{\partial \theta} = -M \cos \theta i_s i_r
\]
ENERGY CONVERSION BETWEEN TWO POINTS

In the $\lambda - i$ plane, to go from $a$ to $b$

$$W_m(\lambda_b, x_b) - W_m(\lambda_a, x_a) = \int_{\lambda_a}^{\lambda_b} id\lambda - \int_{x_a}^{x_b} f^e \, dx$$

$$= \int_{\lambda_a}^{\lambda_b} id\lambda + \left[ -\int_{x_a}^{x_b} f^e \, dx \right]$$

$$\Delta W_{m_{a \rightarrow b}} = EFE|_{a \rightarrow b} + EFM|_{a \rightarrow b}$$

Where EFE stands for “energy from electrical” and EFM stands for “energy from mechanical.” To evaluate EFE and EFM, we need to specify a particular path.
ENERGY CONVERSION OVER A CYCLE

Energy balance

\[ dW_m = id \lambda - \int f^e \, dx \]

Over complete cycle

\[ dW_m = 0 \text{ (system returns to original state)} \]

\[ 0 = \oint i \, d\lambda - \oint f^e \, dx \]

\[ = \oint i \, d\lambda + (-\oint f^e \, dx) \]

\[ id \lambda = \text{Energy from electrical (EFE)} \]

\[ -f^e \, dx = \text{Energy from mechanical (EFM)} \]
ENERGY CONVERSION OVER A CYCLE

Over complete cycle

\[ \int EFE + \int EFM = 0 \quad \text{or} \quad EFE \bigg|_{cycle} + EFM \bigg|_{cycle} = 0 \]

Because there is no net change in stored energy, one can compute either \( EFE \bigg|_{cycle} \) or \( EFM \bigg|_{cycle} \). If \( EFE \bigg|_{cycle} > 0 \), then the system is operating as a \textit{motor} and \( EFM \bigg|_{cycle} < 0 \). If \( EFM \bigg|_{cycle} > 0 \), then the system is operating as a \textit{generator} and \( EFE \bigg|_{cycle} < 0 \).
EXAMPLE
An electric machine has the relation as shown in the figure below. The relations are

\[ \lambda_1 = L_{11} i_1 + M \cos \theta i_2 \]
\[ \lambda_2 = M \cos \theta i_1 + L_{22} i_2 \]

The machine is operated over the cycle ABCD while \( i_1 = I_0 \).
Find the energy converted from electrical to mechanical form for each cycle. Is this a motor or a generator?
EXAMPLE

Torque of electric origin

\[ W'_m = \frac{1}{2} L_{11} i_1^2 + M \cos \theta i_1 i_2 + \frac{1}{2} L_{22} i_2^2 \]

\[ T^e = \frac{\partial W'_m}{\partial \theta} = -M i_1 i_2 \sin \theta \]

Since it is the \( \theta - i \) plane, we compute

\[ \int_{\theta_1}^{\theta_2} T^e \, d\theta \]

\[ EFM|_{cycle} = -\int_0^{2\pi} T^e \, d\theta = -\left[ \int_0^{\pi} T^e \, d\theta + \int_{\pi}^{2\pi} T^e \, d\theta \right] \]

\[ = \int_0^A -T^e \, d\theta + \int_A^B -T^e \, d\theta + \int_B^C -T^e \, d\theta + \int_C^D -T^e \, d\theta \]

\[ EFM_{cycle} = \int_B^C -T^e \, d\theta = \int_{\pi}^{2\pi} M I_0^2 \sin \theta \, d\theta = -2M I_0^2 \]

Since \( EFM|_{cycle} < 0 \), it is motor