Homework 1

Problem 2.1

Converting both the voltage and current waveforms from their time-domain representation to phasors

\[ v(t) = 100 \cos(377t + 10^\circ) \rightarrow \mathbf{V} = \frac{100}{\sqrt{2}} \angle 10^\circ V \]
\[ i(t) = \cos(377t + 55^\circ) \rightarrow \mathbf{I} = \frac{1}{\sqrt{2}} \angle 55^\circ A, \]

the average power and power factor can be calculated

\[ < p(t) > = P = VI \cos(\theta_v - \theta_i) = \frac{50}{\sqrt{2}} W \]
\[ pf = \cos(\theta_v - \theta_i) = \frac{1}{\sqrt{2}} \text{leading} \]

Problem 2.2

(a)

Using the same procedure as Problem 2.1, the voltage and current phasors are

\[ \mathbf{V} = \frac{100}{\sqrt{2}} \angle 15^\circ V \quad \mathbf{I} = -\frac{10}{\sqrt{2}} \angle (45 - 90)^\circ = \frac{10}{\sqrt{2}} \angle 135^\circ A \]

by applying the fact that \( \cos(\alpha \pm \pi) = -\cos(\alpha) \). The real power into the terminals will be

\[ P = \frac{1000}{2} \cos(-120^\circ) = -250 W \]

(b)

As done previously,

\[ P = VI \cos(\theta_v - \theta_i) \]
\[ = \frac{100}{\sqrt{2}} \times \frac{10}{\sqrt{2}} \cos([-75 - (15 - 90 + 180)]^\circ) = -500 W \]

(c)

\[ P = VI \cos(\theta_v - \theta_i) \]
\[ = 100 \times 10 \cos([-30 - 90 + 60]^\circ) = 500 W \]
\[ pf = \cos(-60^\circ) = 0.5 \text{ leading} \]
(d)

\[ P = VI\cos(\theta_v - \theta_l) \]
\[ = \frac{100}{\sqrt{2}} \cdot \frac{10}{\sqrt{2}} \cos\left((30 - 90) - (120 - 90)\right)^\circ = 0 \text{ W} \]
\[ pf = \cos(-90^\circ) = 0 \text{ leading} \]

(e)

Converting the voltage and current phasors from rectangular to polar coordinates yields

\[ \overline{V} = 10 + j10 = 10\sqrt{2}\angle45^\circ V \]
\[ \overline{I} = 5 + j10 = 5\sqrt{5}\angle63.43^\circ A, \]

which can be used to find

\[ P = 50\sqrt{2}\cos((45 - 63.43)^\circ) = 150 \text{ W} \]
\[ pf = \cos(-18.43^\circ) = 0.9487 \text{ leading} \]

(f)

\[ \overline{V} = 100 - j90 = \sqrt{100^2 + 90^2}\angle-41.99^\circ V \]
\[ \overline{I} = 10 - j10 = 10\sqrt{2}\angle-45^\circ A \]
\[ P = 10\sqrt{(100^2 + 90^2) + 2\cos((-41.99 + 45)^\circ)} = 1900 \text{ W} \]
\[ pf = \cos(3.01^\circ) = 0.999 \text{ lagging} \]

(g)

\[ \overline{V} = -10 - j10 = 10\sqrt{2}\angle-135^\circ V \]
\[ \overline{I} = 5 + j10 = 5\sqrt{5}\angle63.43^\circ A \]
\[ P = 50\sqrt{10}\cos((-135 - 63.43)^\circ) = -150 \text{ W} \]

**Problem 2.8**

The total current flowing into the loads is

\[ \overline{I} = \overline{I}_1 + \overline{I}_2 \]
\[ = (50 - j40) + (50 + j60) \]
\[ = 101.98\angle11.3^\circ A \]

The input complex power is simply

\[ \overline{S} = \overline{V}\overline{I} \]
\[ = 100 \times 101.98 \times \sqrt{2}\angle(45^\circ - 11.3^\circ) \]
\[ = 12 + j8 \text{ kVA} \]

resulting in a power factor of

\[ pf = \cos(33.7^\circ) = 0.832 \text{ lag} \]

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Problem 2.9

The complex power absorbed by the loads are

\[
\begin{align*}
\text{Load1} & : 5(0.8 + j \sin(\cos^{-1} 0.8)) = 4 + j3 \text{ kVA} \\
\text{Load2} & : 3 + j \frac{3 \times \sin(\cos^{-1} 0.9)}{0.9} = 3 + 1.45 \text{ kVA},
\end{align*}
\]

resulting in a total input complex power of

\[
\overline{S} = (4 + j3) \text{ kVA} + (3 + j1.45) \text{ kVA} = 7 + j4.45 \text{ kVA}.
\]

The source current is therefore

\[
\bar{I} = \frac{\overline{S}}{\overline{V}} = 7 + j4.45 \text{ kVA}
\]

Problem 2.10

The complex power absorbed by the three loads are

\[
\begin{align*}
\text{Load1} & : 8 + j6 \text{ kVA} \\
\text{Load2} & : 12 - j16 \text{ kVA} \\
\text{Load3} & : 5 + j10 \text{ kVA},
\end{align*}
\]

therefore the overall complex power for the arrangement will be

\[
\overline{S} = \overline{S}_1 + \overline{S}_2 + \overline{S}_3 = 25 + j0 \text{ kVA}.
\]

With the voltage across the parallel load arrangement known to be 250\(^\circ\) V, the line current can be solved as

\[
\overline{S} = \overline{V}_{\text{load}} \overline{I}_{\text{line}} \rightarrow \overline{I}_{\text{line}} = \left(25 + j0 \text{ kVA} \right)^* = 100 \angle 0^\circ \text{A}.
\]

With the series feeder impedance being 0.1+j1.0 \Omega, the voltage at the source (in phasor notation) can be obtained by applying KVL:

\[
\overline{V}_s = 250\angle0^\circ V + 100\angle0^\circ (0.1 + j1.0)V = 278.6\angle21.04^\circ V
\]

Note that in this case the source voltage does not have an angle of 0\(^\circ\), this owes to the fact that the voltage drop across the parallel load arrangement was chosen to have the global reference angle for the problem. Finally, the time-domain representation of the source voltage and current will be

\[
\begin{align*}
\upsilon_s(t) & = 278.6 \sqrt{2} \cos(120\pi t + 21.04^\circ) \text{ V} \\
i_s(t) & = 100 \sqrt{2} \cos(120\pi t) \text{ A}
\end{align*}
\]

Problem 2.11

The complex power absorbed by the three loads are

\[
\begin{align*}
\text{Load1} & : 125 + 216.5 \text{ kVA} \\
\text{Load2} & : 180 - j135 \text{ kVA} \\
\text{Load3} & : 283 + j100 \text{ kVA},
\end{align*}
\]
therefore the overall complex power and associated power factor of the arrangement will be

\[ S = S_1 + S_2 + S_3 = 588 + j181.5 \, kVA \]

\[ pf = \cos \left( \tan^{-1} \left( \frac{181.5}{588} \right) \right) = 0.955 \text{ lagging} \]

If shunt capacitance were added across the parallel load, real power consumption would remain the same while \( S \) and \( Q_{\text{load}} \) change. With the goal to bring the power factor to 0.8 leading, the amount of VARs required for this adjustment is

\[ (\theta_v - \theta_i)_{\text{new}} = -\cos^{-1}(0.8) = -36.87^\circ \]

\[ S_{\text{new}} = \frac{P}{pf} = 735 \, kVA \]

\[ Q_{\text{new}} = S_{\text{new}} \sin(-36.87) = -441 \, kVA \]

\[ Q_{\text{cap}} = |Q_{\text{old}} - Q_{\text{new}}| = 622.5 \, kVAR \]

**Special Problem 1**

(a) Through the application of KVL, in accordance with Fig. ??, a load voltage of \( 120 \angle 0^\circ \) \( V \) requires a source voltage of:

\[
\overline{V_s} = 2(75 \angle 0^\circ \, A)(0.05 + j0.05 \, \Omega) + 120 \angle 0^\circ \, V = 128 \angle 3.37^\circ \, V
\]

(b) For a load power factor of 0.707 lagging, the necessary source voltage is

\[
\overline{V_s} = 2(75 \angle 0^\circ \, A)(0.05 + j0.05 \, \Omega) + 120 \angle 45^\circ \, V = 131 \angle 45^\circ \, V
\]

(c) For a load power factor of 0 leading, the necessary source voltage is

\[
\overline{V_s} = 2(75 \angle 0^\circ \, A)(0.05 + j0.05 \, \Omega) + 120 \angle -90^\circ \, V = 113 \angle -86.19^\circ \, V
\]
Special Problem 2

(a)
The complex power absorbed by the three loads are

\[ \text{Load 1: } 4.8 + j3.6 \text{ kVA} \]
\[ \text{Load 2: } 4 + j1.94 \text{ kVA} \]
\[ \text{Load 3: } 3.12 + j0 \text{ kVA} , \]

therefore the overall complex power of the arrangement will be

\[ \bar{S} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = 11.92 + j5.54 \text{ kVA} , \]

(b)
The source current is given by

\[ \bar{I} = \frac{\bar{S}}{\bar{V}} = \frac{(11.92 + j5.54 \text{ kVA})}{240 \text{ V}} \]
\[ = 49.67 - j23.08 \text{ A} \]
\[ |\bar{I}| = 54.77 \text{ A} \]

(c)
Since the current load arrangement provides a lagging power factor with a phase angle of 24.93°, additional shunt capacitance is required to obtain a power factor of 0.95 lagging (phase angle of 18.19°). The amount of capacitive VARs is:

\[ (\theta_v - \theta_i)_{\text{new}} = \cos^{-1}(0.95) = 18.19° \]
\[ S_{\text{new}} = \frac{P}{pf} = 12.54 \text{ kVA} \]
\[ Q_{\text{new}} = S_{\text{new}} \sin(18.19) = 3.92 \text{ kVA} \]
\[ Q_{\text{cap}} = Q_{\text{old}} - Q_{\text{new}} = 1.62 \text{ kVAR} \]