Useful information

\[ \sin(x) = \cos(x-90^\circ) \quad \bar{V} = ZI \quad \bar{S} = \bar{V}I^* = P + jQ \quad \bar{S}_{\text{ph}} = \sqrt{3}V_L I_L \angle \theta \]

\[ 0 < \theta < 180^\circ \text{ (lag)} \quad I_L = \sqrt{3}I_{\text{ph}} \text{ (delta)} \quad \bar{Z}_Y = \bar{Z}_A / 3 \quad \mu_0 = 4\pi \cdot 10^{-7} \text{ H/m} \]

\[ -180^\circ < \theta < 0 \text{ (lead)} \quad V_L = \sqrt{3}V_{\text{ph}} \text{ (wye)} \]

ABC sequence has A at zero, B at minus 120 degrees, and C at plus 120 degrees

\[ \int_c \mathbf{H} \cdot d\mathbf{l} = \int_s \mathbf{J} \cdot n da \quad \int_c \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_s \mathbf{B} \cdot n da \quad \mathbf{R} = \frac{1}{\mu \Lambda} \quad MMF = N_i = \phi R \]

\[ \phi = B A \quad \lambda = N\phi = Li \text{ (if linear)} \quad v = d\lambda / dt \quad k = \frac{M}{\sqrt{L_1 L_2}} \quad 1 \text{ hp} = 746 \text{ Watts} \]

\[ v_1 = L_1 \frac{d\phi}{dt} - N_1 \frac{d^2\phi}{dt^2} \]

\[ \frac{v_1}{v_2} = \frac{N_j}{N_k} \quad \frac{N_j \dot{i}_1}{N_k \dot{i}_2} = \frac{N_j}{N_k} \]

\[ i' \Rightarrow \frac{N_i}{N_i} \]
Problem 1. (25 points)

Two loads are connected in parallel across a source whose voltage is $1000\angle 60^\circ$ V. Load 1 is 100 kVA at 0.866 PF lagging and load 2 is 40 kW at 0.5 PF lagging. Find the total complex power, the total current $\bar{I}$ supplied by the source, and the currents in each of the loads. What is the kVAR of the capacitor needed to make the overall PF 0.95 lag? The capacitor is connected across the parallel combination.

\[\bar{I}_1 = \sqrt{\bar{I}_1^*} = 100 \angle \cos^{-1} 0.866 = 100 \angle 60^\circ I_1^*\]
\[\Rightarrow \bar{I}_1 = 100 \angle 30^\circ A\]

Load 2:
\[P_2 = \sqrt{2} I_2 \cos \theta \Rightarrow 40 \angle 0^\circ = 1000 I_2 \angle 0.5\]
\[\Rightarrow |I_2| = 80, \text{ P.F. angle } = 60^\circ - \cos^{-1} 0.5 = 0 \Rightarrow \bar{I}_2 = 80 \angle 0^\circ\]

\[\bar{I} = \bar{I}_1 + \bar{I}_2 = 100 \angle 30^\circ + 80 \angle 0^\circ = 173.9 \angle 16.7^\circ A\]
\[\bar{S} = \sqrt{\bar{I}^*} = 1000 \angle 60^\circ \times 173.9 \angle -16.7^\circ = 126.6 + j 119.3\]
\[= 173.9 \angle 43.3^\circ \text{ kVA}\]

New P.F. is 0.95 \[\Rightarrow \cos \theta_{\text{new}} = 0.95, \ \bar{Q}_{\text{new}} = 18.194^\circ\]

\[\bar{Q}_{\text{new}} = P \tan \theta_{\text{new}} \Rightarrow \bar{Q}_{\text{new}} = 126.6 \tan 18.194^\circ \Rightarrow \bar{Q}_{\text{new}} = 41.61 \text{ kVAR}\]
\[\bar{Q}_{\text{new}} = \bar{Q}_{\text{old}} + \bar{Q}_{\text{cap}} \Rightarrow \bar{Q}_{\text{cap}} = 41.61 - 119.3 = -77.69 \text{ kVAR}\]
Problem 2. (25 points)

A three-phase power system consists of a wye-connected source, 60Hz, connected to two loads in parallel through a transmission line having an impedance of \( Z_{\text{line}} = 0 + j1 \ \Omega \).

The first load is wye connected and draws 10 Amps line current at 0.85 P.F. lagging. The second load is delta connected and consumes 1581.138\(\angle18.434^\circ\) VA. Phase to neutral voltage at the load is \(\frac{300}{\sqrt{3}}\) V.

a) Draw the per-phase equivalent circuit with all given voltages, currents, powers, and impedances labeled. [5 points]
b) Find the voltage a: the source.[10 points]
c) Find the complex power delivered by the source.[4 points]
d) Find the per-phase impedance of the delta connected load.[6 points]
Problem 3. (25 points)

For the figure below, for a current \( i = 6 \) A (assume DC) and gap \( g = 0.1 \) cm, find the magnetic flux density (in Tesla) in the center leg and each of the outer legs and the inductance of the coil. You may neglect fringing and assume the permeability of the iron is infinite. Note that the number of turns is given as 200. The side view of the device is also given on the right.

\[
\begin{align*}
R_1 & = \frac{g}{\mu_0 A_1} = \frac{0.1 \times 10^{-2}}{4\pi \times 10^{-7} \times 6 \times 10^{-4}} = 1326292 \frac{\text{A} \cdot \text{T}}{\text{W} \cdot \text{m}} = R_3 \\
R_2 & = \frac{g}{\mu_0 A_2} = \frac{0.1 \times 10^{-2}}{4\pi \times 10^{-7} \times 12 \times 10^{-4}} = 663146 \frac{\text{A} \cdot \text{T}}{\text{W} \cdot \text{m}} = R_1 / 2 \\
\end{align*}
\]
Problem 4.  (25 points)

A multi-coil magnetic circuit is illustrated in the diagram below. The cross-sectional area of the magnetic core is \( A \) everywhere. The permeability of the magnetic core is infinite. There are two air gaps with lengths \( g_1 \) and \( g_2 \) as shown, and permeability \( \mu_0 \). You may neglect fringing. Label the voltages and currents - you may define the two voltages and two currents any way that you like.

\[
\begin{align*}
R_1/R_3 &= \frac{R_1}{2} \\
\Phi_2 &= 1200 \text{AT} \\
-1200 \text{AT} + R_1 \Phi_2 &= 0 \\
\Phi_2 &= 0.000905 \ \text{wb} \\
\Phi_1 &= \frac{\Phi_2}{2} = 0.0004524 \ \text{wb} \\
B_1 &= B_2 = 0.75 \text{T} \\
B_2 &= 0.75 \text{T} \\
\lambda &= N_2 \Phi_2 = 0.181 \ \mu_0 \ \text{wb} \\
\mu_0 &= L \times 6 \text{Ams} \\
L &= 0.03 \ \text{H}
\end{align*}
\]

a) (5 pts) Put the polarity “dots” on the proper side of each coil
b) (10 pts) Find expressions for \( L_1 \), \( L_2 \), and \( M \) in terms of the parameters
c) (5 pts) Find an expression for the coefficient of coupling in terms of only \( g_1 \) and \( g_2 \)
d) (5 pts) What happens to the coefficient of coupling as \( g_1 \) gets smaller and \( g_2 \) gets bigger?
\[ n_1 = N_1 \frac{di_1}{dt} \]
\[ n_2 = N_2 \frac{d\phi_2}{dt} \]
\[ -N_1 i_1 + \frac{g_1}{\mu_0 A} \phi_1 + \frac{g_2}{\mu_0 A} (\phi_1 + \phi_2) = 0 \]
\[ -N_2 i_2 - \frac{g_1}{\mu_0 A} \phi_1 + N_1 i_1 = 0 \]
\[ \phi_1 = \frac{\mu_0 A N_1 i_1}{g_1} - \frac{\mu_0 A N_2 i_2}{g_1} \]
\[ \phi_2 = \frac{\mu_0 A N_1 i_1}{g_2} - \left( \frac{g_1}{g_2} + 1 \right) \frac{\mu_0 A N_2 i_2}{g_1} \]
\[ n_2 = -\frac{\mu_0 A N_1 N_2 i_2}{g_1} + \left( \frac{g_1}{g_2} + 1 \right) \frac{\mu_0 A N_2^2}{g_1} i_2 \]
(1) \[ k = \frac{46AN_{1/2}}{g_1 \sqrt{4OAN_1^2 + 4OAN_2^2/\left(\frac{g_1}{g_2} + 1\right)}} = \sqrt{\frac{g_1}{g_2} + 1} \]

2) If \( g_1 \) gets smaller and \( g_2 \) gets bigger, \( k \) approaches 1.