ECE 330 (Spring 2016) **Instructors:** A. Flifet and S. Bose

Midterm 1

Duration: 90 minutes Total points: 100

Name: Solution.

Section (Tick one): C (Mon/Wed/Fri) _____ F (Tue/Thu) _____.

Scores (For official use only):

Problem 1:_____/25, Problem 2:_____/25,
Problem 3:_____/25, Problem 4:_____/25. **Total score:**_____/100.

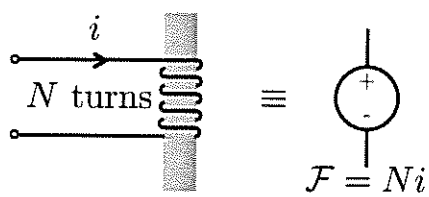
Relevant formulae

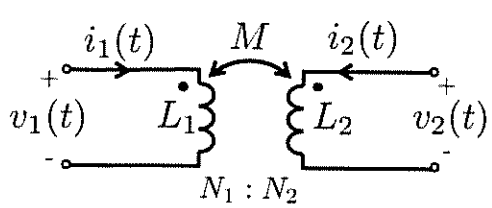
$\sin(x) = \cos(90^o - x)$ $\quad \bar{V} = \bar{I}\bar{Z}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad \bar{S}_{3\phi} = \sqrt{3}V_L I_L \angle\theta$

$\begin{cases} 0^o < \theta < 180^o & \text{: lag} \\ -180^o < \theta < 0^o & \text{: lead} \end{cases}$ $\quad \begin{cases} I_L = \sqrt{3}I_\phi & \text{: delta} \\ V_L = \sqrt{3}V_\phi & \text{: wye} \end{cases}$ $\quad \bar{Z}_Y = \frac{\bar{Z}_\triangle}{3}$ $\quad \mu_0 = 4\pi \times 10^{-7} H/m$

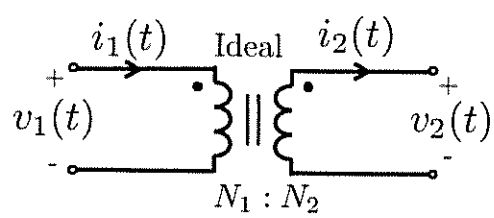
$\int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot \mathbf{n}\, da$ $\quad \int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t}\int_S \mathbf{B} \cdot \mathbf{n}\, da$ $\quad \Re = \frac{l}{\mu A}$ $\quad \mathcal{F}(\text{mmf}) = Ni = \phi\Re$

$\phi = BA$ $\quad \lambda = N\phi$ $\quad v = \frac{d\lambda}{dt}$ $\quad k = \frac{M}{\sqrt{L_1 L_2}}$





$v_1(t) = L_1 \frac{d}{dt}[i_1(t)] + M\frac{d}{dt}[i_2(t)]$



$\frac{i_1(t)}{i_2(t)} = \frac{N_2}{N_1} = \frac{1}{a}$ and $\frac{v_1(t)}{v_2(t)} = \frac{N_1}{N_2} = a$

Problem 1 [25 points]

A single phase voltage source serves three loads connected in parallel at 60 Hz, and 120 V (rms). The three loads are described as follows:

- Load 1 draws 3kVA at 0.6 power factor, lagging.
- Load 2 has an impedance of $(5 + j5)\Omega$.
- Load 3 is inductive, and draws a current of magnitude 10 Amps (rms) and 0.72 kW of real power.

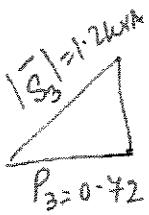
(a) Find the total complex power drawn by all three loads together. [12 points]

(b) Assuming a zero phase angle for the voltage source, compute the current supplied by the source as a function of time. [5 points]

(c) How much reactive VAR should be supplied by a capacitor in parallel to the three loads to make the overall power factor drawn from the source to be unity? What is the capacitance of such a capacitor? [3 + 5 points]

$$\bar{S}_1 = (3\angle 53.2^\circ)\,\text{kVA} = (1.8 + j\,2.4)\,\text{kVA}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}} \Rightarrow \bar{S}_2 = \frac{|\bar{V}|^2}{\bar{Z}^*} = \frac{120^2}{5-j5}\cdot 10^{-3}\ \text{kVA} = (1.44 + j\,1.44)\,\text{kVA}$$



$$|\bar{S}_3| = \frac{120 \times 10}{10^3}\,\text{kVA} = 1.2\ \text{kVA} \Rightarrow \bar{S}_3 = \left[1.2\angle \cos^{-1}\left(\frac{0.72}{1.2}\right)\right]\text{kVA}$$

$$= 1.2\angle 53.2^\circ\ \text{kVA}$$

$$\bar{S}_{Total} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = \left[3\angle 53.2^\circ + (1.44 + j\,1.44) + 1.2\angle 53.2^\circ\right]\text{kVA}.$$

$$= (3.96 + j\,4.80)\,\text{kVA} = (6.22\angle 50.48^\circ)\,\text{kVA}$$

b. $$\bar{V}\bar{I}^* = \bar{S}_{total} \Rightarrow \bar{I} = \left(\frac{\bar{S}_{total}}{\bar{V}}\right)^* = \left[\frac{(6.22\angle 50.48)\times 10^3}{120}\right]^*\ \text{Amps}.$$

$$= 51.83\angle -50.48^\circ\ \text{Amps}.$$

$$= (32.98 - j\,39.98)\ \text{Amps}.$$

$$\therefore i(t) = \mathrm{Re}\{\sqrt{2}\,\bar{I}e^{j\omega t}\} = 73.30\cos(377t - 50.48^\circ).$$

c. $Q_{var} = 4.80$ kVAR. (supplied) $\quad \bar{I}_{cap} = j\bar{V}\omega C$

$$\bar{V}\,\bar{I}_{cap}^{*} = -jQ_{cap} \Rightarrow \bar{V}\cdot\left(\cancel{-j}\,\bar{V}^{*}\omega C\right) = \cancel{-j}\,Q_{cap}$$

$$\Rightarrow C = \frac{Q_{cap}}{|\bar{V}|^{2}\cdot\omega} = \frac{4.80\times10^{3}}{120^{2}\times 2\pi\times 60}$$

$$= 884.2\ \mu F.$$

Problem 2 [25 points]

Suppose a positive sequence three-phase wye-connected generator supplies a delta-connected load through transmission lines that have an impedance of $(1.5\angle 75°)\Omega$ per phase. The line-line voltage across the load is 4.16kV, and the load has a per phase impedance of $(60\angle 30°)\Omega$.

(a) Assuming a zero phase angle for voltage of the source in phase a, draw the per phase equivalent of the circuit for phase a. [5 points]

(b) Compute the current phasors $\bar{I}_a, \bar{I}_b, \bar{I}_c$ that represent the phase currents at the source. Compute the voltage phasors $\bar{V}_{ab}, \bar{V}_{bc}, \bar{V}_{ca}$, that denote the line-line voltages at the source. [5 + 5 points]

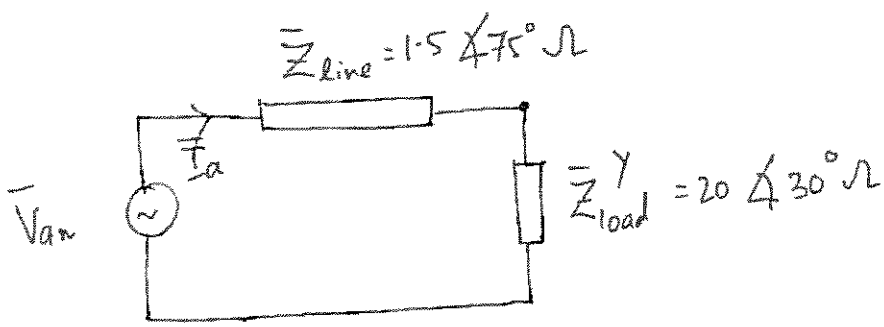
(c) How much total complex power is being drawn by the load? [5 points]

(d) Compute the total power loss in the transmission lines. [5 points]

Transforming the Δ-load to a Y-load, we get

$$\bar{Z}^{Y}_{load} = \bar{Z}^{\Delta}_{load}/3 = 20\angle 30°\ \Omega.$$

a.



Per-phase equivalent circuit.

b. Voltage phasor across load in per-phase diagram

$$= \bar{V}^{\phi}_{Load}.$$

$$|V^{\phi}_{Load}| = V^{line\text{-}line}_{load}/\sqrt{3}.$$

$$= 4.16\ kV/\sqrt{3}$$

$$= 2.4\ kV.$$

Using KVL, we have

① $\bar{I}_a \cdot \bar{Z}^Y_{load} = \bar{V}^{\phi}_{load}$

and

② $\bar{I}_a \cdot \left(\bar{Z}^Y_{load} + \bar{Z}_{line} \right) = \bar{V}_{an}$.

Also, we know $\angle \bar{V}_{an} = 0$.

<u>Solving for $\bar{I}_a$.</u>

- $|\bar{I}_a| = |\bar{V}^{\phi}_{load}| \Big/ |\bar{Z}^Y_{Load}| = 2.4\times10^3 / 20$ Amps

$= 120$ Amps.

- $\angle \bar{I}_a = -\angle \left(\bar{Z}^Y_{load} + \bar{Z}_{Line} \right) \quad [\because \angle \bar{V}_{an} = 0]$.

$= -\angle \left(1.5 \angle 75^\circ + 20 \angle 30^\circ \right)$

$= -\angle \left(17.70 + j\,11.45 \right)$

$= -32.88^\circ$.

$\therefore \bar{I}_a = 120 \angle -32.88^\circ$ Amps

$\bar{I}_b = 120 \angle (-32.88^\circ - 120^\circ)$ Amps

$= 120 \angle -152.88^\circ$ Amps.

$\bar{I}_c = 120 \angle (-32.88^\circ + 120^\circ)$ Amps

$= 120 \angle 87.12^\circ$ Amps.

<u>Solving for $\bar{V}_{ab}$.</u>

$|\bar{V}_{an}| = |\bar{I}_a| \cdot \underbrace{\left| \bar{Z}^Y_{Load} + \bar{Z}_{line} \right|}_{= 21.08 \angle -32.88^\circ} = 120 \times 21.08$ Volts

$= 2.53$ KV.

$$\therefore \bar{V}_{ab} = |\bar{V}_{an}| \cdot \sqrt{3} \cdot \angle 30^\circ \qquad \left[\because \angle \bar{V}_{ab} = 30^\circ + \underbrace{\angle V_{an}}_{=0^\circ}\right]$$

$$= 4.38 \angle 30^\circ \text{ Volts.}$$

$$\bar{V}_{bc} = 4.38 \angle (30^\circ - 120^\circ) \text{ Volts} = 4.38 \angle -90^\circ \text{ Volts}$$

$$\bar{V}_{ca} = 4.38 \angle (30^\circ + 120^\circ) \text{ Volts} = 4.38 \angle 150^\circ \text{ Volts.}$$

c. Complex power drawn by load

$$= \sqrt{3} \underbrace{\left|\bar{V}^{\text{Line-Line}}_{\text{Load}}\right|}_{= 4.16 \text{ kV (given).}} \cdot \underbrace{\left|\bar{I}^{\text{Line-Line}}_{\text{Load}}\right|}_{= |\bar{I}_a| = 120 \text{ Amps}} \cdot \underbrace{\angle(\theta)}_{= 30^\circ}$$

angle of $\bar{Z}^{Y}_{Load}$

$$\cancel{= 8.65 \times 10^5 \text{ VA} = 865 \text{ KVA}}.$$

$$= \cancel{7.4688 \times 10^5} \; 8.64 \times 10^5 \; \angle 30^\circ \text{ VA}$$

$$\cancel{8.65 \times 10^5}$$

d. Power loss in transmission lines

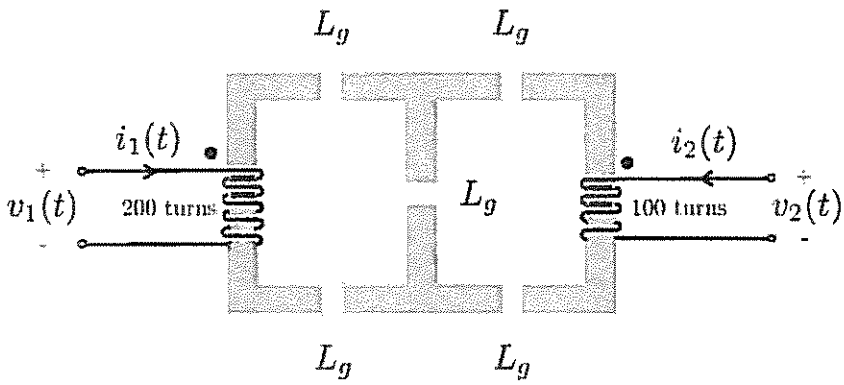
$$= 3 \times \left(|\bar{I}_a|^2 \cdot \underbrace{R_{line}}_{= Re\{Z_{line}\}}\right).$$

$$= 3 \times 120^2 \times 0.3882 \text{ W}$$

$$= 1.67 \times 10^4 \text{ W.}$$

Problem 3 [25 points]

In the circuit shown below, assume the iron core has infinite permeability, i.e., $\mu = \infty$. Neglect fringing effects in the air gaps. The cross-sectional area of all parts of the core is $A = 10\text{cm}^2$. Assume that the windings have no resistance, and neglect any core losses. The air gaps have lengths $L_g = 1\text{mm}$. The permeability of space is given by $\mu_0 = 4\pi \times 10^{-7}\text{H/m}$.



(a) Find the reluctance of each air gap. [2 points]

(b) Draw the magnetic circuit for the iron core and the two windings. [5 points]

(c) Utilize the magnetic circuit to solve for the flux linkages $\lambda_1(t)$ and $\lambda_2(t)$ in the two coils in terms of $i_1(t)$ and $i_2(t)$. [8 points]

(d) Find the self inductances L_1 and L_2, and the mutual inductance M of the two coils. Report the absolute values of each quantity. [1+1+1 points]

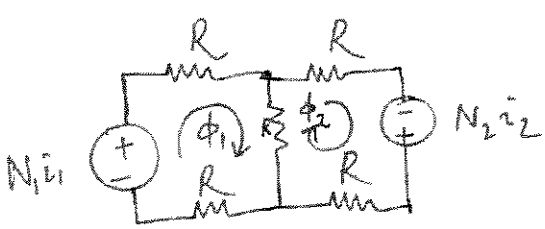
(e) Draw the polarity markings on the coils to indicate the nature of the coupling among the two coils on the above figure. [3 points]

(f) Write down the equations relating $v_1(t)$ to $i_1(t)$ and $i_2(t)$ and possibly their derivatives. Repeat the same for $v_2(t)$. [2 + 2 points]

a. $R = \dfrac{L_g}{\mu_0 \cdot A} = \dfrac{10^{-3}}{4\pi \times 10^{-7} \times 10 \times 10^{-4}} \cdot A\text{-}t/Wb = 7.96 \times 10^{5}\ A\text{-}t/Wb.$

(c)

b.



$N_1 i_1 = \phi_1 R + (\phi_1 - \phi_2)R + \phi_1 R$

$N_2 i_2 = \phi_2 R + (\phi_2 - \phi_1)R + \phi_2 R$

$$\Rightarrow \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \frac{1}{R} \begin{pmatrix} N_1 i_1 \\ N_2 i_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \frac{1}{8R} \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} N_1 i_1 \\ N_2 i_2 \end{pmatrix}$$

$$\phi_1 = \frac{3}{8R} N_1 i_1 + \frac{1}{8R} N_2 i_2$$

and $$\phi_2 = \frac{1}{8R} N_1 i_1 + \frac{3}{8R} . N_2 i_2 .$$

$$\Rightarrow \lambda_1 = \frac{3N_1^2}{8R} i_1 + \frac{N_1 N_2}{8R} i_2$$

$$= 0.0188\, i_1 + 0.0031\, i_2$$

$$\lambda_2 = \frac{N_1 N_2}{8R} i_1 + \frac{3N_2^2}{8R} i_2$$

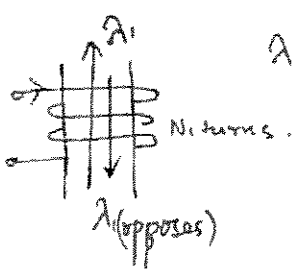
$$= 0.0031\, i_1 + 0.0047\, i_2 .$$

d.

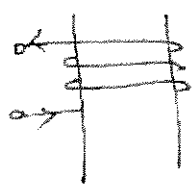
$$L_1 = 0.0188 \text{ H}, \quad L_2 = 0.0047 \text{ H}, \quad M = 0.0031 \text{ H}.$$

e. Marked on the diagram.

f.

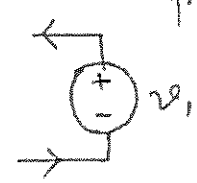


$\lambda_1 \uparrow \Rightarrow \lambda_1$, opposes $\uparrow \Rightarrow$



Direction of current intended through magnetic field induced emf.

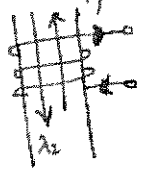
$\equiv$



$$\therefore v_1 = \frac{d\lambda_1}{dt} = 0.0188 \frac{di_1}{dt} + 0.0031 \frac{di_2}{dt} .$$

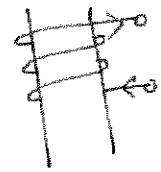
P.T.O.

λ_2, opposes.



$\lambda_2 \uparrow \Rightarrow \lambda_2$, opposes $\uparrow$

$\Rightarrow$ current direction through emf should look like

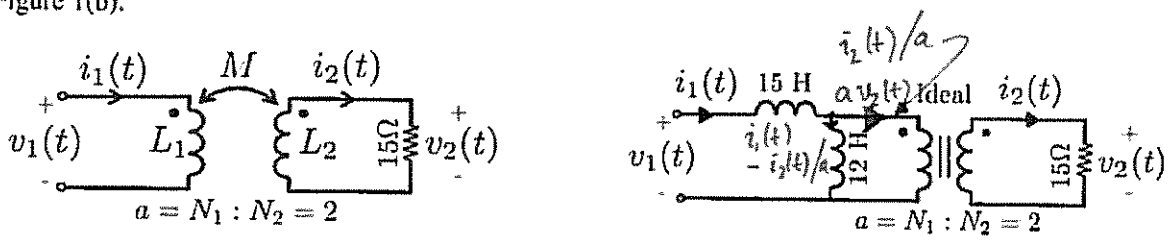


$\Rightarrow$ emf should look like

$$\therefore v_2 = \frac{d\lambda_2}{dt} = 0.0031 \frac{di_1}{dt} + 0.0047 \frac{di_2}{dt}.$$

Problem 4 [25 points]

A transformer with self-inductances L_1, L_2, and mutual inductance M supplies a load of 15Ω connected across the second coil. The coupled coil representation for the transformer is given in Figure 1(a). Suppose the turns ratio is $a = N_1 : N_2 = 2$. The equivalent circuit for the transformer and the load is shown in Figure 1(b).



(a) Coupled coil representation

(b) Equivalent circuit representation

Figure 1: A transformer supplying a load.

(a) Write down the "loop equations", i.e., Kirchhoff's voltage laws, for the coupled coil representation shown in Figure 1(a) to obtain relationships among the variables $v_1(t)$, $i_1(t)$, $v_2(t)$, $i_2(t)$ in terms of L_1, L_2, and M. [6 points]

(b) Write down the "loop equations" for the equivalent circuit representation shown in Figure 1(b). [6 points]

(c) The circuits in Figures 1(a) and 1(b) are meant to represent the same transformer. By equating the loop equations you derived in parts (a) and (b), find L_1, L_2, and M. [2 + 2 + 2 points]

(d) If $v_1(t) := 100\sqrt{2}\cos(t)$V is supplied across the first coil, compute the voltage phasor $\bar{V}_2$ across the load. Assume zero phase angle for the source. [7 points].

a. $v_1(t) = L_1 \cdot \frac{d}{dt} i_1(t) + M \frac{d}{dt}\left(-i_2(t)\right)$.

and $v_2(t) = L_2 \cdot \frac{d}{dt}\left(-i_2(t)\right) + M \frac{d}{dt}\left(i_1(t)\right)$.

Also, $v_2(t) = i_2(t) \cdot (15\Omega)$

$\Rightarrow i_2(t) \cdot (15\Omega) = L_2 \cdot \frac{d}{dt}\left(-i_2(t)\right) + M \frac{d}{dt}\left(i_1(t)\right)$.

b. $v_1 - (15H) \cdot \frac{d i_1(t)}{dt} - (12H) \cdot \frac{d}{dt}\left(i_1(t) - \frac{i_2(t)}{a}\right) = 0$.

$v_2(t) = (15\Omega) \cdot (i_2(t))$, and $av_2(t) = (12H) \cdot \frac{d}{dt}\left(i_1(t) - \frac{i_2(t)}{a}\right)$.

$$\Rightarrow (15\Omega)\cdot i_2(t) = \frac{12H}{a}\frac{d}{dt}\left(i_1(t) - i_2(t)/a\right).$$

c. Coefficient of $\frac{di_1}{dt}$ and $\frac{di_2}{dt}$ in loop equation for v_1 must be the same

$$\Rightarrow L_1 = 15H + 12H = 27H.$$

~~Coefficient~~ and

$$-M = \frac{-12H}{2} = -6H$$

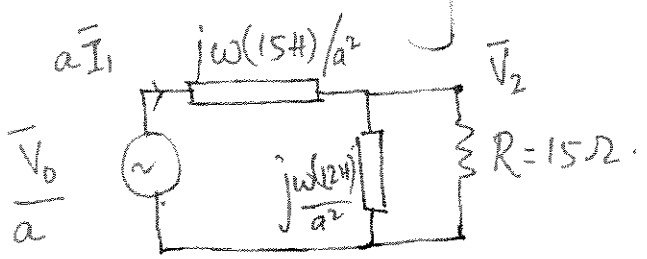
$$\Rightarrow M = 6H.$$

Similarly for the equations with v_2.

$$-L_2 = -\frac{1}{a^2}(12H)$$

$$\Rightarrow L_2 = 3H.$$

d. Referring the primary side circuit to the secondary side, we get.



$$a\bar{I}_1 = \frac{\bar{V}_0/a}{\underbrace{\bar{Z}_{eq}}_{= j\omega(15H)/a^2 + \left[R \,\|\, \frac{j\omega(12H)}{a^2}\right]}}$$

$\omega = 1.$

$$\Rightarrow \bar{Z}_{eq} = j\frac{15}{4}\,\Omega + (15 \,||\, j3)\,\Omega$$

$$= \left[0.5769 + j\,6.6346\right]\,\Omega$$

$$\therefore\ a\bar{I}_1 = \frac{100\angle 0^\circ/2}{0.5769 + j\,6.6346}\ \text{Amps.}$$

$$\therefore\ \bar{V}_2 = \frac{\bar{V}_0}{a} - (a\bar{I}_1)\cdot j\frac{\omega\,(15H)}{a^2}$$

$$= \frac{100\angle 0^\circ}{2} - \frac{100\angle 0^\circ/2}{0.5769 + j\,6.6346}\cdot j\frac{15}{4}\ \text{Volts.}$$

$$= 50\left(1 - \frac{j\,15/4}{0.58 + j\,6.63}\right)\ \text{Volts.}$$

$$= 21.95 - j2.44\ \text{Volts.}$$

$$= 22.09\angle -6.34^\circ\ \text{Volts.}$$

Other ways to solve it: ~~use~~ the coupled coils.