ECE 330 (Spring 2016)  

**Midterm 1**  

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Duration: 90 minutes  

Total points: 100

**Name:**

Section (Tick one): C (Mon/Wed/Fri) F (Tue/Thu)

Scores (For official use only):  

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**Relevant formulae**

\[
\sin(x) = \cos(90^\circ - x) \quad \quad \quad \quad V = \bar{I}\bar{Z} \quad \quad \quad S = \bar{V}\bar{I}^* \quad \quad \quad S_{3\phi} = \sqrt{3}V_L I_L \angle \theta
\]

\[
\begin{align*}
0^\circ < \theta < 180^\circ : \text{lag} & \quad \quad \quad I_L = \sqrt{3}I_\phi : \text{delta} \\
-180^\circ < \theta < 0^\circ : \text{lead} & \quad \quad \quad V_L = \sqrt{3}V_\phi : \text{wye}
\end{align*}
\]

\[
\int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot n \, da \quad \quad \int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot n \, da
\]

\[
f = \frac{1}{\mu_0} \quad \quad \quad \quad \mathbf{F}(\text{mmf}) = Ni = \phi\mathbf{N}
\]

\[
\phi = BA \quad \quad \quad \lambda = N\phi \quad \quad \quad v = \frac{d\lambda}{dt} \quad \quad \quad k = \frac{M}{\sqrt{L_1L_2}}
\]

\[
\begin{align*}
\mathbf{i} & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad N \text{ turns} \\
\mathbf{F} & \quad \quad \quad \quad \quad \quad \quad \quad = Ni
\end{align*}
\]

\[
v_1(t) = L_1 \frac{d}{dt} [i_1(t)] + M \frac{d}{dt} [i_2(t)]
\]

\[
\begin{align*}
\frac{i_1(t)}{i_2(t)} = \frac{N_2}{N_1} = \frac{1}{a} \quad \text{and} \quad \frac{v_1(t)}{v_2(t)} = \frac{N_1}{N_2} = a
\end{align*}
\]
Problem 1 [25 points]

A single phase voltage source serves three loads connected in parallel at 60 Hz, and 120 V (rms). The three loads are described as follows:

- Load 1 draws 3kVA at 0.6 power factor, lagging.
- Load 2 has an impedance of (5 + j5)Ω.
- Load 3 is inductive, and draws a current of magnitude 10 Amps (rms) and 0.72 kW of real power.

(a) Find the total complex power drawn by all three loads together. [12 points]
(b) Assuming a zero phase angle for the voltage source, compute the current supplied by the source as a function of time. [5 points]
(c) How much reactive VAR should be supplied by a capacitor in parallel to the three loads to make the overall power factor drawn from the source to be unity? What is the capacitance of such a capacitor? [3 + 5 points]

\[
\begin{align*}
\overline{S}_1 &= (3 \angle 53.2°) \text{ kVA} = (1.8 + j2.4) \text{ kVA} \\
\overline{I}_2 &= \frac{\overline{V}}{Z} \Rightarrow \overline{S}_2 = \frac{|\overline{V}|^2}{Z^*} = \frac{120^2}{5-j5} \cdot 10^{-3} \text{ kVA} = (1.44 + j1.44) \text{ kVA} \\
|\overline{S}_3| &= 120 \times 10^3 \text{ kVA} = 1.2 \text{ kVA} \Rightarrow \overline{S}_3 = \left[ 1.2 \angle \cos^{-1} \left( \frac{0}{1.2} \right) \right] \text{ kVA} \\
&= 1.2 \angle 53.2° \text{ kVA} \\
\overline{S}_{total} &= \overline{S}_1 + \overline{S}_2 + \overline{S}_3 = \left[ 3 \angle 53.2° + (1.44+j1.44) + 1.2 \angle 53.2° \right] \text{ kVA} \\
&= (3.96 + j4.80) \text{ kVA} = (6.22 \angle 50.48°) \text{ kVA} \\
\overline{I}^* &= \overline{S}_{total} \Rightarrow \overline{I} = \left( \frac{\overline{S}_{total}}{\overline{V}} \right)^* = \left( \frac{6.22 \angle 50.48° \times 10^3}{120} \right)^* \\
&= 51.83 \angle -50.48° \text{ Amps} \\
&= (32.98 - j39.78) \text{ Amps} \\
\end{align*}
\]

\[\text{Output: } Q_{\text{var}} = 4.80 \text{ kVAR} \text{ (Supplied)}, \quad I_{\text{Cap}} = jV \text{ Amperes}.\]
\[ \bar{V} \bar{I}_{\text{cap}}^* = j \Phi_{\text{cap}} \Rightarrow \bar{V} \cdot (j\bar{V} \omega \overline{C}) = -j \Phi_{\text{cap}} \]

\[ \Rightarrow C = \frac{\Phi_{\text{cap}}}{|\bar{V}|^2 \omega} = \frac{4 \times 10^3}{120^2 \times 2 \pi \times 60} = 88.4 \ \mu F. \]
Problem 2 [25 points]

Suppose a positive sequence three-phase wye-connected generator supplies a delta-connected load through transmission lines that have an impedance of \((1.5 \angle 75^\circ) \Omega\) per phase. The line-line voltage across the load is 4.10kV, and the load has a per phase impedance of \((60 \angle 30^\circ) \Omega\).

(a) Assuming a zero phase angle for voltage of the source in phase \(a\), draw the per phase equivalent of the circuit for phase \(a\). [5 points]

(b) Compute the current phasors \(I_a, I_b, I_c\) that represent the phase currents at the source. Compute the voltage phasors \(V_{ab}, V_{bc}, V_{ca}\), that denote the line-line voltages at the source. [5 + 5 points]

(c) How much total complex power is being drawn by the load? [5 points]

(d) Compute the total power loss in the transmission lines. [5 points]

Transforming the \(\Delta\)-load to a \(Y\)-load, we get:
\[
\overline{Z}_\text{load} = \overline{Z}_\text{load} / 3 = 20 \angle 30^\circ \Omega.
\]

\[
\overline{Z}_\text{line} = 1.5 \angle 75^\circ \Omega
\]

\[
\overline{Z}_\text{load} = 20 \angle 30^\circ \Omega
\]

\[
\text{b: Voltage phasor across load in per-phase diagram}
\]
\[
|V_\Phi| = \frac{V_{\text{line-line}}}{\sqrt{3}} = 4.16 \frac{kV}{\sqrt{3}} = 2.4 \text{ kV}.
\]
Using KVL, we have

1. \( \bar{I}_a \cdot \bar{Z}_{\text{load}} = \bar{V}_{\text{load}} \)

and

2. \( \bar{I}_a \cdot (\bar{Z}_{\text{load}} + \bar{Z}_{\text{line}}) = \bar{V}_{\text{an}} \).

Also, we know \( \Delta \bar{V}_{\text{an}} = 0 \).

Solving for \( \bar{I}_a \).

\[
|\bar{I}_a| = \frac{|\bar{V}_{\text{load}}|}{|\bar{Z}_{\text{load}}|} = \frac{2.4 \times 10^3}{20} \text{ Amps} = 120 \text{ Amps}.
\]

\[
\Delta \bar{I}_a = - \Delta \left( \bar{Z}_{\text{load}} + \bar{Z}_{\text{line}} \right) \quad \left[\because \Delta \bar{V}_{\text{an}} = 0 \right].
\]

\[
= - \Delta \left( 1.5 \Delta 75^\circ + 20 \Delta 30^\circ \right)
\]

\[
= - \Delta \left( 17.70^\circ + j11.45 \right)
\]

\[
= -32.88^\circ.
\]

\(\therefore \) \( \bar{I}_a = 120 \Delta -32.88^\circ \text{ Amps} \)

\( \bar{I}_b = 120 \Delta (-32.88^\circ -120^\circ) \text{ Amps} = 120 \Delta -152.88^\circ \text{ Amps} \).

\( \bar{I}_c = 120 \Delta (-32.88^\circ + 120^\circ) \text{ Amps} = 120 \Delta 87.12^\circ \text{ Amps} \).

Solving for \( \bar{V}_{an} \).

\[
|\bar{V}_{an}| = |\bar{I}_a| \cdot \left| \bar{Z}_{\text{load}} + \bar{Z}_{\text{line}} \right|^6 = 120 \times 21.08 \text{ Volts} = 253 \text{ kV}.
\]
\[ V_{ab} = |V_{an}| \cdot \sqrt{3} \cdot \Delta 30^\circ = 4.38 \Delta 30^\circ \text{ Volts}. \]

\[ V_{bc} = 4.38 \Delta (30^\circ - 120^\circ) \text{ Volts} = 4.38 \Delta -90^\circ \text{ Volts}. \]

\[ V_{ca} = 4.38 \Delta (30^\circ + 120^\circ) \text{ Volts} = 4.38 \Delta 150^\circ \text{ Volts}. \]

\[ \text{Complex power drawn by load} = \sqrt{3} \left( V_{line-line} \right) \text{ Volts} \times |I_{load}| \text{ Amps}. \]

\[ = 4.16 \text{ kV} \text{ (given)} \]

\[ = 6.65 \times 10^5 \text{ VA} = 865 \text{ kVA}. \]

\[ = 8.64 \times 10^5 \text{ \Delta } 30^\circ \text{ VA}. \]

\[ \text{Power loss in transmission lines} = 3 \times (|I_{al}|^2 \cdot R_{line}). \]

\[ = R \{ Z_{line} \}. \]

\[ = 3 \times 120^2 \times 0.3882 \text{ W}. \]

\[ = 1.67 \times 10^4 \text{ W}. \]

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Problem 3 [25 points]

In the circuit shown below, assume the iron core has infinite permeability, i.e., $\mu = \infty$. Neglect fringing effects in the air gaps. The cross-sectional area of all parts of the core is $A = 10\text{cm}^2$. Assume that the windings have no resistance, and neglect any core losses. The air gaps have lengths $L_g = 1\text{mm}$. The permeability of space is given by $\mu_0 = 4\pi \times 10^{-7}\text{H/m}$.

\[ \begin{align*}
L_g & \quad L_g \\
+ & \quad + \\
| & \quad | \\
\text{i}_1(t) & \quad \text{i}_2(t) \\
\text{v}_1(t) & \quad \text{v}_2(t) \\
200 \text{ turns} & \quad 100 \text{ turns} \\
- & \quad - \\
L_g & \quad L_g \\
\text{v}_1(t) & \quad \text{v}_2(t) \\
- & \quad - \\
L_g & \quad L_g \\
\end{align*} \]

(a) Find the reluctance of each air gap. [2 points]

(b) Draw the magnetic circuit for the iron core and the two windings. [5 points]

(c) Utilize the magnetic circuit to solve for the flux linkages $\lambda_1(t)$ and $\lambda_2(t)$ in the two coils in terms of $i_1(t)$ and $i_2(t)$. [8 points]

(d) Find the self inductances $L_1$ and $L_2$, and the mutual inductance $M$ of the two coils. Report the absolute values of each quantity. [1+1+1 points]

(e) Draw the polarity markings on the coils to indicate the nature of the coupling among the two coils on the above figure. [3 points]

(f) Write down the equations relating $\text{v}_1(t)$ to $\text{i}_1(t)$ and $\text{i}_2(t)$ and possibly their derivatives. Repeat the same for $\text{v}_2(t)$. [2+2 points]

\[ a. \quad R = \frac{L_g}{\mu_0 \cdot A} = \frac{10^{-3}}{4\pi \times 10^{-7} \times 10 \times 10^{-4}} \quad \text{A} \cdot \text{t}/\text{Wb} = 7.96 \times 10^{-5} \quad \text{A} \cdot \text{t}/\text{Wb} \]

\[ b. \quad N_1 \text{i}_1 = \phi_1 \cdot R + (\phi_1 - \phi_2) \cdot R + \phi_1 \cdot R \]

\[ N_2 \text{i}_2 = \phi_2 \cdot R + (\phi_2 - \phi_1) \cdot R + \phi_2 \cdot R \]

\[ \Rightarrow \quad \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \frac{1}{R} \begin{pmatrix} N_1 \text{i}_1 \\ N_2 \text{i}_2 \end{pmatrix} \]

\[ \Rightarrow \quad \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \frac{1}{8R} \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} N_1 \text{i}_1 \\ N_2 \text{i}_2 \end{pmatrix} \]
\[ \phi_1 = \frac{3 N_1 i_1}{8R} + \frac{1}{8R} N_2 i_2 \]

and \[ \phi_2 = \frac{1}{8R} N_1 i_1 + \frac{3}{8R} N_2 i_2 \].

\[ \Rightarrow \lambda_1 = \frac{3N_1^2}{8R} i_1 + \frac{N_1 N_2}{8R} i_2 \]

\[ = 0.0188 i_1 + 0.0031 i_2 \]

\[ \lambda_2 = \frac{N_1 N_2}{8R} i_1 + \frac{3N_2^2}{8R} i_2 \]

\[ = 0.0031 i_1 + 0.0047 i_2 \].

\( d. \)

\[ L_1 = 0.0188 \, H, \quad L_2 = 0.0047 \, H, \quad M = 0.0031 \, H. \]

e; Marked on the diagram.

\( f. \)

\[ \lambda_1 \uparrow \Rightarrow \lambda_1, \text{ opposes } \uparrow \Rightarrow \]

\[ \text{Direction of current intended through magnetic field induced emf.} \]

\[ = \]

\[ \text{\onslide<2>} \]

\[ v_1 = \frac{d\lambda_1}{dt} = 0.0188 \frac{di_1}{dt} + 0.0031 \frac{di_2}{dt}, \]

P.T.O.
\[ \lambda_2 \Rightarrow \lambda_2, \text{ opposed} \uparrow \]

\Rightarrow \text{ current direction through emf}

\Rightarrow \text{ emf should look like}

\[ \text{emf should look like} \]

\[ V_2 = \frac{d\lambda_2}{dt} = 0.0031 \frac{d}{dt} i_1 + 0.0047 \frac{d}{dt} i_2. \]
Problem 4 [25 points]

A transformer with self-inductances $L_1$, $L_2$, and mutual inductance $M$ supplies a load of 15Ω connected across the second coil. The coupled coil representation for the transformer is given in Figure 1(a). Suppose the turns ratio is $a = N_1 : N_2 = 2$. The equivalent circuit for the transformer and the load is shown in Figure 1(b).

(a) Write down the “loop equations”, i.e., Kirchhoff’s voltage laws, for the coupled coil representation shown in Figure 1(a) to obtain relationships among the variables $v_1(t)$, $i_1(t)$, $v_2(t)$, $i_2(t)$ in terms of $L_1$, $L_2$, and $M$. [6 points]

(b) Write down the “loop equations” for the equivalent circuit representation shown in Figure 1(b). [6 points]

(c) The circuits in Figures 1(a) and 1(b) are meant to represent the same transformer. By equating the loop equations you derived in parts (a) and (b), find $L_1$, $L_2$, and $M$. [2 + 2 + 2 points]

(d) If $v_1(t) := 100\sqrt{2}\cos(t)V$ is supplied across the first coil, compute the voltage phasor $\tilde{V}_2$ across the load. Assume zero phase angle for the source. [7 points]

\[ a. \quad v_1'(t) = L_1 \cdot \frac{d}{dt} i_1(t) + M \frac{d}{dt} (-i_2(t)) \]

\[ \text{and} \quad v_2'(t) = L_2 \cdot \frac{d}{dt} (-i_2(t)) + M \frac{d}{dt} (i_1(t)). \]

Also,

\[ v_2'(t) = i_2(t) \cdot (15\Omega) \]

\[ \Rightarrow \quad i_2(t) \cdot (15\Omega) = L_2 \cdot \frac{d}{dt} (-i_2(t)) + M \frac{d}{dt} (i_1(t)). \]

\[ b. \quad v_1 - (15H) \cdot \frac{d}{dt} i_1(t) - \left[ (12H) \cdot \frac{d}{dt} \left( i_1(t) - \frac{i_2(t)}{a} \right) \right] = 0. \]

\[ v_2'(t) = (15\Omega) \cdot i_2(t), \quad \text{and} \quad a \cdot v_2'(t) = (12H) \cdot \frac{d}{dt} \left( i_1(t) - \frac{i_2(t)}{a} \right). \]
\[ (15 \pi \cdot t \cdot t) \cdot i_2(t) = \frac{24 \cdot t}{a} \cdot \frac{d}{dt} \left( i_1(t) - i_2(t) / a \right). \]

Coefficient of \( \frac{di}{dt} \) and \( \frac{di_2}{dt} \) in loop equation for \( V_1 \) must be the same.

\[ L_1 = 15 \, \text{H} + 24 \, \text{H} = 27 \, \text{H}. \]

Coefficient and

\[ M = \frac{-24 \, \text{H}}{2} = -12 \, \text{H} \]

\[ M = 6 \, \text{H}. \]

Similarly for the equations with \( V_2 \).

\[ L_2 = \frac{-1}{a^2} (24 \, \text{H}) \]

\[ L_2 = 3 \, \text{H}. \]

Referring the primary side circuit to the secondary side, we get:

\[ a \bar{I}_1 = \frac{\bar{V}_b}{a} \]

\[ Z_{eq} = j \omega (15 \, \text{H}) / a^2 + \left[ R \left( \frac{j \omega (15 \, \text{H})}{a} \right) \right]^{-1} \]
\[ \omega = 1. \]

\[ \Rightarrow Z_{eq} = j \frac{15}{4} \omega + (15 \angle 3) \omega \]

\[ = [0.5769 + j 6.6346] \omega \]

\[ \therefore aI_1 = \frac{100 \angle 40^\circ / 2}{0.5769 + j 6.6346} \text{ Amps}. \]

\[ \therefore V_2 = \frac{V_0 - (aI_1) \cdot j \omega (15H)}{a^2} \]

\[ = \frac{100 \angle 0^\circ}{2} - \frac{100 \angle 40^\circ / 2}{0.5769 + j 6.6346} \cdot j \frac{15}{4} \text{ Volts}. \]

\[ = 50 \left( 1 - \frac{j \frac{15}{4}}{0.58 + j 6.63} \right) \text{ Volts}. \]

\[ = 21.95 - j 2.44 \text{ Volts}. \]

\[ = 22.09 \angle -6.34^\circ \text{ Volts}. \]

Other ways to solve it: ESE the coupled coils.