- Homeworks are due Tuesdays at 5 p.m. Late homework will not be accepted.
- Unstapled homeworks will not be accepted.
- Write your name, netID, and section on each homework.
- Homework are to be turned in to homework boxes on 3rd floor of ECEB. Please put your homework into the appropriate box for the section you are registered in:
  - Section X - 12:00 PM (Xu Chen): Box 35
  - Section E - 1:00 PM (Lynford Goddard): Box 36
  - Section F - 2:00 PM (Yang Zhao): Box 37
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions. If you use any source outside of class materials that we've provided, you must cite every source that you used.
- Use of homework solutions from past semesters is not allowed and is considered cheating. Copying homework solutions from another student is considered cheating.
- Penalties for cheating on homework: 50% reduction in homework average on first offense, 100% reduction in homework average and report to college on second offense.

Reading Assignment: Kudeki: Lectures 21-24
Recommended Reading: Staelin: 2.3, 2.5-2.7

1. For each of the three electromagentic waves described in (a), (b), and (c), find the requested parameter:

   a) $\mu = \mu_o$ and $\varepsilon = 12\varepsilon_o$, electric field linearly polarized in the $\hat{x}$ direction, poynting vector in the $-\hat{y}$ direction, electric field amplitude E_o, radial frequency $\omega_o = 12\pi \times 10^{14}$rad/s:

      i. Impedance of the medium and the speed of wave propagation.

      ii. Wavelength (λ) and Wavevector (β) for the wave. Is this wave visible to the human eye? If so, what color would we associate with this wave? If not, what wavelength range does it fall into?

      iii. Write the Electric and Magnetic fields as cosinusoidal monochromatic waves $\mathbf{E}(\mathbf{r}, t)$, $\mathbf{H}(\mathbf{r}, t)$

      iv. Write the Electric and Magnetic fields in phasor notation $\tilde{E}$, $\tilde{H}$.

      v. Give the complex Poynting Vector $\tilde{S}$ and time averaged Poynting Vector $\langle \mathbf{S} \rangle$.

   b) $\mu = \mu_o$ and $v = c/2$, magnetic field linearly polarized in the $\hat{z}$ direction, electric field in the $\hat{y}$ direction, magnetic field amplitude H_o, wavelength in the medium $\lambda = 1\mu$m:

      i. Impedance of the medium and the material permittivity.

      ii. Radial frequency (ω) and Waveperiod (T) for the wave. Is this wave visible to the human eye? If so, what color would we associate with this wave? If not, what wavelength range does it fall into?

      iii. Write the Electric and Magnetic fields as cosinusoidal monochromatic waves $\mathbf{E}(\mathbf{r}, t)$, $\mathbf{H}(\mathbf{r}, t)$

      iv. Write the Electric and Magnetic fields in phasor notation $\tilde{E}$, $\tilde{H}$.

      v. Give the complex Poynting Vector $\tilde{S}$ and time averaged Poynting Vector $\langle \mathbf{S} \rangle$.

   c) $\varepsilon = \varepsilon_o$ and $v = c$, right-hand circularly polarized, propagating in the $+\hat{z}$ direction, electric field amplitude E_o, frequency $f = 3$GHz:

      i. Impedance of the medium and the material permeability.

      ii. Wavelength (λ) and wavevector (β) for the wave. Is this wave visible to the human eye? If so, what color would we associate with this wave? If not, what wavelength range does it fall into?

      iii. Write the Electric and Magnetic fields as cosinusoidal monochromatic waves $\mathbf{E}(\mathbf{r}, t)$, $\mathbf{H}(\mathbf{r}, t)$

      iv. Write the Electric and Magnetic fields in phasor notation $\tilde{E}$, $\tilde{H}$.

      v. Give the complex Poynting Vector $\tilde{S}$ and time averaged Poynting Vector $\langle \mathbf{S} \rangle$.

2. Consider an infinite surface current density

$$\mathbf{J_s} = \hat{x} J_{so} \sin(\omega t)$$

   flowing on $z = 0$ surface, where $J_{so} > 0$ is real-valued amplitude of the monochromatic surface current measured in A/m units. It is found that $\mathbf{J}_s$ injects field energy into propagating transverse electromagnetic (TEM) waves away from the $z = 0$ plane at an average rate of 4 W/m^2 — that is, the magnitude of the average Poynting vector $\langle \mathbf{S} \rangle$ is 2 W/m^2 for the waves excited above and below the surface current.

a) Denoting the TEM waves excited by $\mathbf{J}_s$ (above and below the $z = 0$ plane) as $\mathbf{E} = -\hat{x}E_o \sin(\omega t \mp \beta z)$ V/m and $\mathbf{H} = \mp\hat{y}H_o \sin(\omega t \mp \beta z)$ A/m, where wavenumber $\beta = \frac{\omega}{c}$, determine the numerical values of wave amplitudes E_o and H_o in V/m and A/m units (assuming wave propagation in free space).

b) Determine the value of J_{so} in A/m units.

c) Write down explicitly the phasors of **E** and **H** fields of part (a) (for regions above and below $z = 0$ plane) using the numerical values of E_o and H_o determined in that part.

3. In general conducting media, plane TEM wave parameters γ and η satisfy

$$\gamma\eta = j\omega\mu \text{ and } \frac{\gamma}{\eta} = \sigma + j\omega\epsilon,$$

as well as

$$\mu = \frac{\gamma\eta}{j\omega}, \quad \sigma = \mathrm{Re}\{\frac{\gamma}{\eta}\}, \quad \epsilon = \frac{1}{\omega}\mathrm{Im}\{\frac{\gamma}{\eta}\}.$$

Using these relations, for a plane wave propagating in a non-magnetic material ($\mu = \mu_o$) with

$$\mathbf{H} = \hat{x}\, 25e^{-z} \cos(8\pi \cdot 10^6 t - \sqrt{3}z - \frac{\pi}{3}) \frac{\mathrm{A}}{\mathrm{m}},$$

Determine:

a) The propagation constant γ and intrinsic impedance η,

b) The permittivity ϵ and conductivity σ,

c) Phasor magnetic field $\tilde{\mathbf{H}}$,

d) The corresponding phasor electric field $\tilde{\mathbf{E}}$,

e) Time-averaged Poynting vector $\langle \mathbf{E} \times \mathbf{H} \rangle$,

f) The time averaged power dissipated in cubic volume bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$, all in meters.

4. A submarine submerged in the ocean ($\sigma = 4$ S/m, $\epsilon_r = 81$, $\mu_r = 1$) wants to receive an electromagnetic signal from a ship located at the surface and transmitting at $f = \frac{\omega}{2\pi} = 90$ kHz.

a) Calculate the numerical values of the propagation constant $\gamma = \alpha + j\beta$ and intrinsic impedance η. Use the approximations appropriate for the associated loss tangent $\frac{\sigma}{\omega\epsilon}$.

b) How close by must the submarine be located in order to receive at least 1% of the signal amplitude at the surface?

c) What is the wavelength $\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f}$ of the EM signal at this operating frequency?

d) Repeat parts (a-c) for ship transmission at 900 Hz.

5. An infinite plane current sheet of uniform, time-varying density $\mathbf{J_s}(t) = 2\cos(6\pi \times 10^8 t)\hat{y}$ A/m exists at the $x = 2$ m plane within a perfect dielectric medium having an electric permittivity of $\epsilon = \frac{9}{4}\epsilon_0$ and a magnetic permeability of $\mu = \mu_0$. Answer the following questions (using appropriate units) about the plane TEM waves which will propagate away from this surface current source.

a) What is the magnitude and direction of the wave propagation velocity $\mathbf{v_p}$ of the TEM wave through the dielectric medium?

b) What is the wave number β and wavelength λ of the TEM wave?

c) What is the intrinsic impedance η of the dielectric medium?

d) Write the expressions for wavefields $\mathbf{E}$ and $\tilde{\mathbf{E}}$ (the phasor of $\mathbf{E}$) and $\mathbf{H}$ and $\tilde{\mathbf{H}}$ (the phasor of $\tilde{\mathbf{H}}$) in terms of ω, β, and η for each of the regions $x > 2$ and $x < 2$ on either side of the current sheet source.

e) Verify that the TEM wave satisfies the Poynting theorem, $\nabla \cdot (\mathbf{E} \times \mathbf{H}) + \frac{\partial}{\partial t}(\frac{1}{2}\epsilon\mathbf{E} \cdot \mathbf{E} + \frac{1}{2}\mu\mathbf{H} \cdot \mathbf{H}) + \mathbf{J} \cdot \mathbf{E} = 0$, in the region $x > 2$. **Hint**: you should be able to prove this for a TEM wave propagating at an arbitrary frequency ω in an arbitrary perfect dielectric in terms of μ and ϵ in order to avoid explicit use of constants found in parts (a-c).

f) What is the time-averaged power transported by the TEM wave across a square surface on the $x = 4$ m plane having area $A = 2\ \mathrm{m}^2$?

g) Verify that the *time-averaged* Poynting theorem is satisfied at $x = 2$ (i.e., at the location of the current sheet).

6. **Bonus Problem** An infinitely conducting metallic region resides starting on the $z = z_0$ plane and extends into the positive z direction. For each of the following waves, identify 1) the polarization direction, 2) the propagation direction, and 3) $\hat{k}\cdot\mathbf{E}$. Using your knowledge of boundary conditions, describe (in words and diagrams) what happens to the following electric fields when they reach the boundary coming from from $z < z_0$. ***Hint**: Consider transverse and normal components of the electric field. If the field has more than one vector component, you may treat each independently. The component of field propagating in the $\hat{x}$-direction does not interact with the surface.* You will learn more about *oblique incidence* in ECE 350.

   a) $\tilde{\mathbf{E}} = \hat{y}E_0 e^{-j\frac{k}{\sqrt{2}}(x+z)}\mathrm{V/m}$

   b) $\tilde{\mathbf{E}} = \frac{(\hat{x}-\hat{z})}{\sqrt{2}}E_0 e^{-j\frac{k}{\sqrt{2}}(x+z)}\mathrm{V/m}$