- Homeworks are due Tuesdays at 5 p.m. Late homework will not be accepted.
- Unstapled homeworks will not be accepted.
- Write your name, netID, and section on each homework.
- Homework are to be turned in to homework boxes on 3rd floor of ECEB. Please put your homework into the appropriate box for the section you are registered in:
  - Section X - 12:00 PM (Xu Chen): Box 35
  - Section E - 1:00 PM (Lynford Goddard): Box 36
  - Section F - 2:00 PM (Yang Zhao): Box 37
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions. If you use any source outside of class materials that we've provided, you must cite every source that you used.
- Use of homework solutions from past semesters is not allowed and is considered cheating. Copying homework solutions from another student is considered cheating.
- Penalties for cheating on homework: 50% reduction in homework average on first offense, 100% reduction in homework average and report to college on second offense.

Reading Assignment: Kudeki: Lectures 20-22
Recommended Reading: Staelin: 2.3, 2.5-2.7

1. A z-polarized plane TEM wave is propagating in vacuum (i.e., $v = c \approx 3 \times 10^8$ m/s and $\eta = \eta_o \approx 120\pi\,\Omega$) in $-x$ direction. If in the wave field varies at $x = 0$ with time according to $E_z(0,t) = 2\triangle(\frac{t}{\tau})$ V/m, where $\tau = 3\,\mu$s and $\triangle(\frac{t}{\tau})$ is the unit triangle function with width τ,

   a) Determine the vector wavefield $\mathbf{E}(x,t)$ written explicitly in terms of all space and time variables x and t,

   b) Determine the accompanying wavefield $\mathbf{H}(x,t)$ in A/m units,

   c) Determine the maximum value of *Poynting vector* $\mathbf{E} \times \mathbf{H}$.

   d) What trajectory function $x = x(t)$ describes instantaneous locations of the peak of $\mathbf{E} \times \mathbf{H}$.

   e) Plot $E_z(x,t)$ vs t at $x = -1000$ m.

   f) Plot $H_y(x,t)$ vs x at $t = 20\,\mu$s.

2. In a homogeneous lossess dielectric with $\epsilon = \epsilon_r\epsilon_o$ and $\mu = \mu_r\mu_o$ a plane TEM wave with the following components is observed:

$$\mathbf{E} = \hat{x}u(t - \frac{z}{c/2}) + \hat{y}g(t - \frac{z}{c/2})\,\frac{\mathrm{V}}{\mathrm{m}}$$

   and

$$\mathbf{H} = \hat{x}(\frac{40z}{c} - 20t) + \hat{y}\frac{1}{60\pi}u(t - \frac{2z}{c})\,\frac{\mathrm{A}}{\mathrm{m}},$$

   where $u(t)$ denotes the unit-step function and c is the speed of light in free space. Using the above information,

   a) Determine the intrinsic impedance η for the medium.

   b) Determine the propagation velocity v.

   c) Determine ϵ_r and μ_r.

   d) Function $g(t)$.

3. Consider a pulse of sheet current $\mathbf{J}_s(t) = \hat{y}\,2t\ \mathrm{rect}(\frac{t}{\tau})$ A/m, where $\tau = 2\,\mu$s on the $z = 0$ plane embedded in free space and $rect(\frac{t}{\tau})$ is the unit rectangular function with width τ defined as:

$$rect(\frac{t}{\tau}) = \begin{cases} 0 & for\ |t| > \frac{\tau}{2} \\ 1 & for\ |t| < \frac{\tau}{2} \end{cases}$$

   The sheet will generate TEM waves which will propagate away from the current source in the $\pm\hat{z}$ directions.

   a) Write the expressions for $\mathbf{E}(z,t)$ and $\mathbf{H}(z,t)$.

   b) Plot the field components for $\mathbf{E}(z,t)$ and $\mathbf{H}(z,t)$ as a function of t at $z = 1500$ m.

   c) Repeat (b) for $z = -1500$ m.

   d) Plot the field components for $\mathbf{E}(z,t)$ and $\mathbf{H}(z,t)$ as a function of z for $t = 6\,\mu$s.

   e) Determine the TEM wave energy radiated per unit area (in J/m^2 units) by the current pulse. **Hint:** integrate the power injected per unit area, $-\mathbf{J}_s \cdot \mathbf{E}$, over the duration of pulse $\mathbf{J}_s(t)$.

4. Given a uniform TEM plane wave with $\mathbf{E}(z,t) = \cos(\omega t + \beta z)\,\hat{y}\,\frac{\text{V}}{\text{m}}$ propagating through a homogeneous perfect dielectric having $\epsilon = 3\epsilon_o$ and phase speed $v_p = \frac{2}{3}c$ and frequency $f = 300\text{MHz}$, determine the following in appropriate units:

    a) the relative permeability μ_r and the intrinsic impedance η of the medium
    b) the wavelength λ of the wave
    c) the phase constant β
    d) the corresponding **H** wavefield
    e) the instantaneous Poynting vector at $t = 10\text{ns}$ on the $z = \frac{1}{2}$ m plane
    f) the instantaneous Poynting vector at $t = 10\text{ns}$ on the $z = 1$ m plane
    g) in view of energy conservation and the results of parts (e) and (f), determine if energy is being stored into or released from the EM field at $t = 10\text{ns}$, within a cube of length $\frac{1}{2}$m located between the $z = \frac{1}{2}$ m and the $z = 1$ m planes.

5. For each of the following plane TEM waves in free space:

    a) $\mathbf{E}_1 = 10\cos(\omega t - \beta x)\hat{y}$ V/m
    b) $\mathbf{H}_2 = 5\cos(\omega t + \beta z + \frac{\pi}{3})\hat{x} + 5\sin(\omega t + \beta z - \frac{\pi}{6})\hat{y}$ A/m
    c) $\mathbf{H}_3 = \sin(\omega t - \beta z + \frac{\pi}{4})\hat{x} - 2\sin(\omega t - \beta z - \frac{3\pi}{4})\hat{y}$ A/m
    d) $\mathbf{E}_4 = 2\cos(\omega t + \beta y - \frac{\pi}{2})\hat{x} - 2\sin(\omega t + \beta y)\hat{z}$ V/m
    e) $\mathbf{H}_5 = \cos(\omega t - \beta y)\hat{x} + \sin(\omega t - \beta y - \frac{\pi}{4})\hat{z}$

        i. Determine the expression for **H** or **E** that accompanies the given wave field
        ii. Find the corresponding field phasors $\tilde{\mathbf{E}}$ **and** $\tilde{\mathbf{H}}$ for all wave fields
        iii. Find the expression for the instantaneous power that crosses a 1 m^2 area in the direction of the wave propagation
        iv. Find the time-averaged power that crosses a 1 m^2 area in the direction of the wave propagation. Hint: you should be able to answer this question based on either part (ii) or (iii)

6. **Bonus Problem** Two plane waves propagate in free space and are described by $\mathbf{E}_1 = E_0\cos(\omega_1 t - \beta_1 z)\hat{x}$ V/m and $\mathbf{E}_2 = E_0\cos(\omega_2 t - \beta_2 z)\hat{x}$V/m where the ω's are at optical frequencies. The two waves simultaneously illuminate an optical detector at some position along z. An optical detector converts power into current and the current produced on that detector is therefore proportional to $\mid \mathbf{E}_1 + \mathbf{E}_2 \mid^2$. Determine the frequencies of the currents that are produced at the detector.