ECE 329 **Homework 6** **Due: Tuesday, February 26, 2019, 5PM**

- Homeworks are due Tuesdays at 5 p.m. Late homework will not be accepted.
- Unstapled homeworks will not be accepted.
- Write your name, netID, and section on each homework.
- Homework are to be turned in to homework boxes on 3rd floor of ECEB. Please put your homework into the appropriate box for the section you are registered in:
  - Section X - 12:00 PM (Xu Chen): Box 35
  - Section E - 1:00 PM (Lynford Goddard): Box 36
  - Section F - 2:00 PM (Yang Zhao): Box 37
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions. If you use any source outside of class materials that we've provided, you must cite every source that you used.
- Use of homework solutions from past semesters is not allowed and is considered cheating. Copying homework solutions from another student is considered cheating.
- Penalties for cheating on homework: 50% reduction in homework average on first offense, 100% reduction in homework average and report to college on second offense.

Reading Assignment: Kudeki: Lectures 14-17
Recommended Reading: Staelin: 2.1-2.2, 2.4-2.6, 3.2-3.4, 4.3

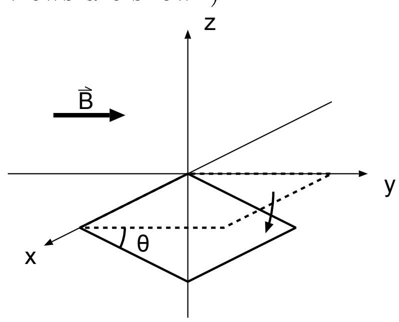
1. Given the time-varying magnetic field $\mathbf{B} = B_0(t\sin(\omega t)\ \hat{y} - \cos(\omega t)\ \hat{z})$ Wb/m^2, find the emf $\mathcal{E}$ for the following closed paths C:

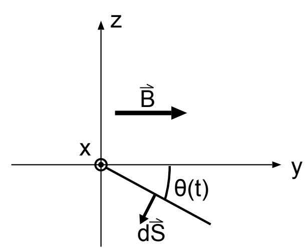
   a) C is a rectangular path going from $(0,0,0)$ to $(0,1,0)$ to $(1,1,0)$ to $(1,0,0)$ back to $(0,0,0)$, with distance along the path measured in units of meters.

   b) C is a rectangular path having the same coordinates as defined above, but with the path direction reversed.

   c) C is a triangular path going from $(0,0,0)$ to $(1,0,0)$ to $(1,1,0)$ back to $(0,0,0)$, with distance along the path measured in units of meters.

2. A square loop of wire of some finite resistance R and 4 cm^2 surface area is located within a region of constant magnetic field $\mathbf{B} = 8\hat{y}$ Wb/m^2 as illustrated in the following diagrams (perspective and side views are shown).





   a) What is the magnetic flux Ψ through the loop when the orientation angle of the loop is $\theta = 0^o$? When the orientation angle of the loop is $\theta = 90^o$? In your flux calculation make use of **dS** orientation shown in the diagram on the right.

   b) What is the flux Ψ as a function of angle θ (using the same sign convention as in part a)?

   c) Assuming that angle θ is time varying at a rate of $\frac{d\theta}{dt} = \pi\,\frac{\text{rad}}{\text{s}}$, and $d\mathbf{S}$ is pointing in the $-\hat{z}$ at time $t = 0$s. What is the emf $\mathcal{E}$ around the loop at $t = 0.25$s?

   d) In what direction will a positive induced current flow around the loop at the same instant? You may draw a picture to explain your answer. Be sure to justify your answer.

   e) What is the emf $\mathcal{E}$ derived from using the **opposite dS** orientation to that shown in the figure? In what direction will a postive induced current flow around the loop in this case?

3. A conducting wire loop of radius $r = 1$ m is moved with velocity $\mathbf{v} = 2\hat{x}$ m/s in a region where the background magnetostatic field is described by

$$\mathbf{B}(x, y, z) = \hat{z}\,25 \times 10^{-6}(1 - x/L)\,\mathrm{T},$$

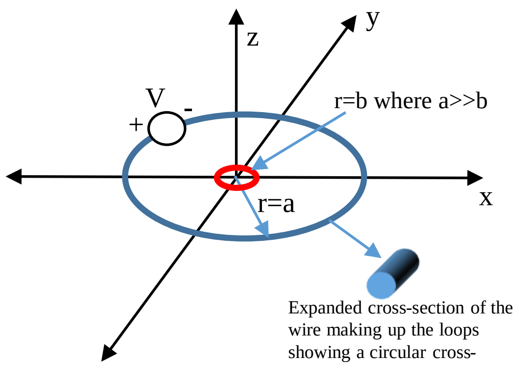
   where $L = 1000$ m. The center of the loop coincides with the origin $(x, y, z) = (0, 0, 0)$ at $t = 0$ and the plane of the loop coincides with $z = 0$ plane.

   a) Obtain an expression for the induced emf $\mathcal{E}(t)$ of the loop in motion for $t > 0$. Since $r \ll L$, the magnetic field across the loop can be considered nearly constant at each instant in time.

   b) What is the magnitude of the loop current as a function of time for $t > 0$ if the loop resistance is $2\,\Omega$?

Interesting facts: The strength of Earth's magnetic field is just about 25×10^{-6} T at equatorial latitudes. However, the scale length L associated with the spatial variation of Earth's magnetic field is much longer than 1000 m.

4. Consider two concentric circular wire loops of radii $a = 8$ cm and $b = 0.5$ cm placed on the $x - y$ plane of the reference coordinate system with their centers at the origin. The medium is free space. The conductivity of the wire from which both loops are made is $\sigma = 4 \times 10^7$ S/m. The cross-section of the wire is circular with radius $r_w = 1$ mm and a 3 V battery is connected in the outer loop (see figure).



a) Calculate the current I_a that flows in the outer loop. **Hint:** resistance R of the outer loop can be calculated using the expression for R developed in Lecture 10 in terms of $d = 2\pi a$ and $A = \pi r_w^2$.

b) Derive an expression for the magnetic flux $\Psi_{a\to b}$ that links the inner loop due to the current flowing in the outer loop. Your expression should be in terms of the magnetic permeability of free space, the current I_a, and the radii of the two loops. **Hint:** Refer to the Lecture 13 notes for expressions for the magnetic flux density due to a current flowing in a circular wire loop. Also, take advantage of the fact that $b \ll a$ so that the magnetic field across the smaller loop can be considered nearly constant.

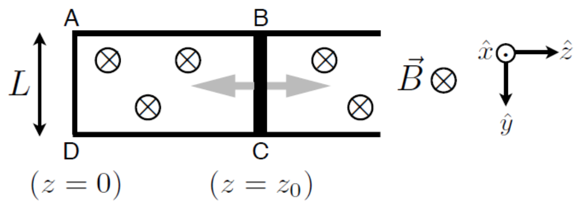
c) $L_{a\to b} \equiv \Psi_{a\to b}/I_a$ is *defined* to be the **mutual inductance** between the outer and inner loop. What is the numerical value of the mutual inductance $L_{a\to b}$?

d) Assume next that the inner loop is moving upwards, in the positive z direction, with speed 2 m/s. Calculate the *emf* induced in the inner loop *assuming* that the flux linking the loop due to the induced current is negligible.

e) You are given that the inductance of the inner loop is 0.2 μH. If the loop is moving upwards with speed 2 m/s, derive a differential equation for the induced current in the loop.

   Hint: the current flowing around the inner loop is the induced emf of the loop divided by the loop resistance while the emf is the negative of the time derivative of the total magnetic flux produced by the currents flowing in both loops — constructing an expression for the current amounts to finding the differential equation that is requested.

5. As shown in the diagram below, a pair of conducting rails separated by a distance L is connected at $z = 0$ to a fixed conducting rod (AD) and at $z = z_0$ to a conducting armature (BC) that can slide along the rail in the $\pm\hat{z}$ direction. A constant magnetic field $\mathbf{B} = -B_0\,\hat{x}$ exists in the region, which is shown pointing down into the page in the diagram. The armature is mechanically pulled in the $+\hat{z}$ direction at a constant velocity $\mathbf{v} = v_0\hat{z}$ m/s from its starting position at $z = z_0$, and the changing magnetic flux through the loop ABCD induces an electromotive force $\mathcal{E}$ and thus a current I_0 around the conducting loop.



a) What is the emf $\mathcal{E}$ induced in the loop ABCD?

b) What is the magnitude and direction of the induced current I_0 in terms of the resistance R of the conducting loop ABCD?

c) What is the magnitude and direction of the Lorentz force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ that is exerted on the armature by the magnetic field $\mathbf{B}$? **Hint**: express your answer in terms of the induced current I_0.

d) Now consider that the armature is no longer moved mechanically (though it is still free to move in response to Lorentz forces) and that a projectile of mass M is attached to it. A constant current of magnitude I' is introduced into the loop at point A such that it flows along the contour ABCD. The magnetic field generated by this current loop is negligibly small compared to the background magnetic field $\mathbf{B}$, and the mass of the armature is negligibly small compared to M. What is the magnitude and direction of the acceleration $\mathbf{a}$ of the projectile? **Hint**: $\mathbf{F} = M\mathbf{a}$.

6. **Bonus Problem:** Faraday's law describes how a changing magnetic field induces an electric field in much the same way (circulation relation) that Ampere's Law describes how a current creates a magnetic field. We can even use some of the same "tricks" when calculating these integrals. Consider a uniform magnetic field $\mathbf{B}(t) = \sin(\omega t)\hat{z}$.

a) What is the divergence of the induced electric field? (In fact, the divergence of ANY induced field)

b) In which direction will the induced electric field point?

c) Calculate the induced electric field as a function of distance r from the origin. Note that this electric field exists, even though there are no wire loops or charges present.

d) Repeat this calculation, except now use the magnetic field produced by an infinite wire located along the $\hat{z}$ axis that is carrying a current given by $\mathbf{I}(z) = \sin(\omega t)\hat{z}$ A.

e) Something about the result to part (d) should bother you – can you see it? What approximation did you have to make to solve part (d)? Why isn't this a problem for part (c)?