- Homeworks are due Tuesdays at 5 p.m. Late homework will not be accepted.
- Unstapled homeworks will not be accepted.
- Write your name, netID, and section on each homework.
- Homework are to be turned in to homework boxes on 3rd floor of ECEB. Please put your homework into the appropriate box for the section you are registered in:
  - Section X - 12:00 PM (Xu Chen): Box 35
  - Section E - 1:00 PM (Lynford Goddard): Box 36
  - Section F - 2:00 PM (Yang Zhao): Box 37
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions. If you use any source outside of class materials that we've provided, you must cite every source that you used.
- Use of homework solutions from past semesters is not allowed and is considered cheating. Copying homework solutions from another student is considered cheating.
- Penalties for cheating on homework: 50% reduction in homework average on first offense, 100% reduction in homework average and report to college on second offense.

Reading Assignment: Kudeki: Lectures 6-9
Recommended Reading: Staelin: 2.4-2.6, 3.1, 4.5, 5.2

1. Consider a simplified model of a **vacuum diode** consisting of a **cathode** in the $x = 0$ plane and an **anode** in the $x = d$ plane, where the anode is held to a constant potential $V_a = 2$ V relative to the cathode. The region $0 < x < d$ between the cathode and the anode supports a charge density $\rho(x)$ accounting for the electrons in transit from the cathode (where they are emitted) to the anode. If the potential distribution in the region $0 < x < d$ is given by $V(x) = V_a(\frac{x}{d})^{4/3}$, find the following:

   a) What is the velocity of an electron $v_e(\frac{x}{d})$ as a function of position in the range $0 < x < d$? Hint: remember what $V(x)$ means, physically.

   b) Plot the electric field $E_x(x)$,

   c) Plot the volumetric charge density $\rho(x)$,

   d) Determine the surface charge density ρ_s on the anode.

2. Given that $V(x, y, z) = x^2 - yz$ and $\mathbf{E} = -\nabla V$, what is $\nabla \times \mathbf{E}$?

3. An important vector identity which is true for any vector field $\mathbf{A}(x, y, z)$ is

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A},$$

   where

$$\nabla^2 \mathbf{A} \equiv (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})\mathbf{A}$$

   is the *Laplacian* of $\mathbf{A}$ and $\nabla(\nabla \cdot \mathbf{A})$ is the gradient of the divergence of $\mathbf{A}$.

   Verify the identity for $\mathbf{A} = (x - y)\,\hat{x} + (x + y)\,\hat{y} + 2\,\hat{z}$ by calculating each side of the identity and showing them to be the same.

4. In a given region, $\mathbf{E} = 3x\,\hat{x} + y\,\hat{y} + 2z^2\,\hat{z}\ \frac{\mathrm{V}}{\mathrm{m}}$.

   a) Determine the electrostatic potential $V(1, 2, 3)$ if $V(0, 0, 0) = 0$.

   b) What is the charge density ρ at the points $(0, 0, 0)$ and at $(1, 2, 3)$?

5. Given the fields $\mathbf{E_1} = y\hat{x} - x\hat{y}\ \frac{\mathrm{V}}{\mathrm{m}}$ and $\mathbf{E_2} = y\hat{x} + x\hat{y}\ \frac{\mathrm{V}}{\mathrm{m}}$, determine the circulation $\oint_C \mathbf{E} \cdot d\mathbf{l}$ for both $\mathbf{E_1}$ and $\mathbf{E_2}$ along a triangular path C traversing in order its vertices at $(x, y, z) = (-1, -1, 0)$, $(-1, 1, 0)$, and $(1, 1, 0)$ m.

   Hint: $d\mathbf{l} = (-\hat{x} - \hat{y})\,dx$ and $x = y$ on the slant edge of C.

6. Consider a static volumetric charge density $\rho(x, y, z) = 6\delta(y) + \rho_s\delta(y - 6)\ \frac{\mathrm{C}}{\mathrm{m}^3}$ in a given region of free space (having permittivity ϵ_0), where the displacement field is $\mathbf{D} = \hat{x}\,2\epsilon_0 + \hat{y}\,4\epsilon_0\ \frac{\mathrm{C}}{\mathrm{m}^2}$ for $0 < y < 6$ m and $D_y = 2\epsilon_0\ \frac{\mathrm{C}}{\mathrm{m}^2}$ for $y > 6$ m. Furthermore, field $\mathbf{D}$ is uniform in each of regions $y < 0$, $0 < y < 6$ m, and $y > 6$ m.

   a) Determine ρ_s,

   b) Determine $\mathbf{D}$ for the region $y > 6$ m,

   c) Determine $\mathbf{D}$ for the region $y < 0$.

   d) Determine $\mathbf{E}$ in all three regions ($y < 0$, $0 < y < 6$, and $y > 6$)

   e) What is the voltage drop from the $y = 0$ m plane to the $y = 6$ m plane?

f) Which of your answers above (parts a, b, c, d, or e) would change if the region from $0 < y < 6$ m were filled with a perfect dielectric having permittivity $\epsilon = 4\epsilon_0$ instead of a vacuum having ϵ_0? Explain.

7. **Bonus Problem:** A volume of charge resides in some randomly distributed region of space, and produces a potential with some distribution $V(x, y, z)$. We have a sensitive piece of equipment that resides at position (x', y', z'), and we wish to shield it from the electric fields that produce the scalar potential. Explain in detail how this could be done. Justify why your solution works using Maxwell's Equations, sketches, etc. A one-line answer will not suffice for full credit.