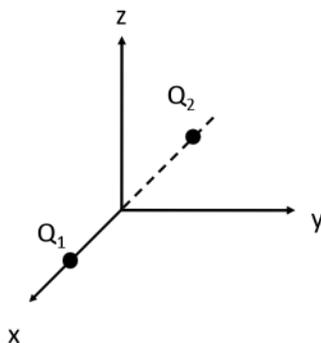


- Homeworks are due Tuesdays at 5 p.m. Late homework will not be accepted.
- Unstapled homeworks will not be accepted.
- Write your name, netID, and section on each homework.
- Homework are to be turned in to homework boxes on 3rd floor of ECEB. Please put your homework into the appropriate box for the section you are registered in:
 - Section X - 12:00 PM (Xu Chen): Box 35
 - Section E - 1:00 PM (Lynford Goddard): Box 36
 - Section F - 2:00 PM (Yang Zhao): Box 37
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions. If you use any source outside of class materials that we've provided, you must cite every source that you used.
- Use of homework solutions from past semesters is not allowed and is considered cheating. Copying homework solutions from another student is considered cheating.
- Penalties for cheating on homework: 50% reduction in homework average on first offense, 100% reduction in homework average and report to college on second offense.

Reading Assignment: Kudeki: Lectures 3-6

Recommended Reading: Staelin: 1.3, 2.1, 2.4

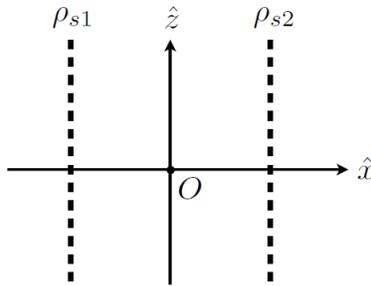
- A particle of mass $m = 36\pi \times 10^{-3}$ kg and charge Q is inserted at time $t = 0$ at a distance $d = 9.8$ m above a planar sheet charged uniformly with electric charge density of $\rho_s = 9.8 \frac{\mu\text{C}}{\text{m}^2}$. The distance d is much smaller than the dimensions of the sheet so that, for all practical purposes the sheet can be assumed to be of infinite extent surrounded by free space.
 - What should the charge Q of the particle be in order for it to levitate motionless at the position where it was placed at $t = 0$? Assume that the Coulomb electric field generated by charge Q is insignificant relative to the electric field generated by the charged sheet (i.e., treat Q as a test charge).
 - Consider, next, the case where $Q = 1$ C. Describe the motion of the particle for $t > 0$ by calculating its acceleration, $a(t)$, its velocity, $v(t)$, and its distance from the charged sheet. For your calculations, use as your reference coordinate system one with its plane $z = 0$ taken to coincide with the plane of the charged sheet.
- Gauss' Law for electric field \mathbf{E} states that $\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\epsilon_0} \int_V \rho dV$ over any closed surface S enclosing a volume V in which electric charge density is specified by $\rho(x, y, z) \frac{\text{C}}{\text{m}^3}$.
 - What is the *electric flux* $\oint_S \mathbf{E} \cdot d\mathbf{S}$ over the surface of a cube of volume $V = L^3$ centered on the origin, if $\rho(x, y, z) = 2 \text{ C/m}^3$ within V and $L = 2$ mm?
 - Repeat (a) for $\rho(x, y, z) = x^2 + y^2 + z^2 \frac{\text{C}}{\text{m}^3}$.
 - What is the *displacement flux* $\oint_S \mathbf{D} \cdot d\mathbf{S}$ over the surface S in part (b)?
 - What is the displacement flux in part (b) for any one of the square surfaces of volume V ?
- Two unknown charges, Q_1 and Q_2 are located at $(x, y, z) = (1, 0, 0)$ and $(-1, 0, 0)$, respectively, as shown below. The displacement flux $\int_{yz\text{-plane}} \mathbf{D} \cdot \hat{x} dy dz$ through the entire yz -plane (i.e., the $x = 0$ plane) in the $+\hat{x}$ direction is -2 C. The displacement flux through the $y = 1$ plane in the $-\hat{y}$ direction is 1 C. Determine Q_1 and Q_2 after writing a pair of algebraic equations relating the above displacement fluxes to Q_1 and Q_2 . **Hint:** What is the contribution of Q_1 to the flux $\int_{xy\text{-plane}} \mathbf{D} \cdot \hat{x} dy dz$? See Example 5 in Lecture 3.



- An infinitely long, cylindrical wire of radius $R = 1$ cm is centered along the \hat{z} axis and carries a uniform volumetric charge density ρ_0 . As with a uniform, infinitesimally thin line charge along \hat{z} (see Example 2 in Lecture 2 online notes), the electric field associated with this charge distribution is

everywhere oriented radially (with no azimuthal or \hat{z} components), owing to the cylindrical symmetry of the charge distribution.

- a) Given that $E_r = -\frac{1}{9\epsilon_0}$ V/m at a distance $r = 3$ cm, use $\mathbf{D} = \epsilon_0\mathbf{E}$ and Gauss' Law with a cylindrical surface of radius $r = 3$ cm and length L to determine the charge density ρ_0 of the wire.
 - b) Calculate the strength of the electric field \mathbf{E} inside the wire, where $r = \frac{1}{2}$ cm. **Hint:** How does the total charge enclosed in a cylindrical Gaussian surface of radius $r < R$ vary with r ?
 - c) Show that the electric field strength is continuous at the surface of the wire, where $r = R$, and sketch the magnitude of the electric field as a function of r for $0 < r < 3R$.
5. Consider two infinite parallel slabs having equal widths W and equal charge densities. Slab 1 is parallel to the xy -plane and extends from $z = -2W$ to $z = -W$ while slab 2 extends from $z = W$ to $z = 2W$. Both slabs have a negative uniform charge density $-\rho_0 C/m^3$. The charge density is zero everywhere else.
- a) Determine and sketch the electric field component \mathbf{E}_z in terms of ρ_0 over the region $-3W < z < 3W$ by using shifted and scaled versions of the static field configuration of a single charged slab (See Example 4 in Lecture 3 online). Be sure to label both axes of your plot and mark the field value at each break point.
 - b) Verify that your field from part (a) satisfies Gauss' Law in differential form, $\nabla \cdot \mathbf{D} = \rho$ in each slab as well as in the free space regions.
6. An infinite sheet that is uniformly charged with a density $\rho_s \frac{C}{m^2}$ produces an electrostatic field \mathbf{E} which has a magnitude $\frac{\rho_s}{2\epsilon_0}$ and points away from the sheet on both sides. Using superposition, determine the charge density ρ_{s2} that is required to produce a displacement field $\mathbf{D} \equiv \epsilon_0\mathbf{E} = 6\hat{x} \frac{C}{m^2}$ at the origin (labeled O on the figure below), if the charge density ρ_{s1} is given as follows:



- a) $\rho_{s1} = 8 \frac{C}{m^2}$
- b) $\rho_{s1} = -\rho_{s2}$

7. **Curl and divergence** exercises:

- a) On a 25-point graph consisting of x and y coordinates having integer values $-2, -1, 0, 1, 2$, sketch the vector field $\mathbf{F} = x\hat{x} + y\hat{y}$ and calculate $\nabla \times \mathbf{F}$ (curl of \mathbf{F}) and $\nabla \cdot \mathbf{F}$ (divergence of \mathbf{F}).
- b) Repeat (a) for $\mathbf{F} = y\hat{x} + 2x\hat{y}$.
- c) (Select one): If $\nabla \times \mathbf{F} \neq 0$, the field strength varies **along** or **across** the direction of the field.

- d) (Select one): If $\nabla \cdot \mathbf{F} \neq 0$, the field strength varies **along** or **across** the direction of the field.
8. Given that $\mathbf{E} = \sin(y)\hat{x} + \cos(x)\hat{y}$ V/m in free space,
- determine $\nabla \times \mathbf{E}$ and $\nabla \times \nabla \times \mathbf{E}$
 - determine ρ such that Gauss' Law is satisfied
9. Given an electrostatic potential $V(x, y, z) = x^2 - 2$ V in a certain region of free space, determine the corresponding
- electrostatic field $\mathbf{E} = -\nabla V$
 - curl $\nabla \times \mathbf{E}$
 - divergence $\nabla \cdot \mathbf{E}$
 - charge density ρ in the region
10. **Bonus Problem:**
- On a 25-point graph consisting of x and y coordinates having integer values -2, -1, 0, 1, 2, sketch the vector field, $\mathbf{D} = \frac{1}{r^2}\hat{r}$ where r is the distance from the origin. You can exclude the origin from your sketch.
 - Judging from your sketch and your understanding of divergence, do you expect $\nabla \cdot \mathbf{D}$ to be positive, negative, or zero?
 - Calculate the divergence $\nabla \cdot \mathbf{D}$. You can use the coordinate transformations $r = \sqrt{x^2 + y^2 + z^2}$ and $\hat{r} = \frac{x\hat{x} + y\hat{y} + z\hat{z}}{\sqrt{x^2 + y^2 + z^2}}$. Is the result consistent with your predictions from part (b)? If not, explain why.