- Homeworks are due Tuesdays at 5 p.m. Late homework will not be accepted.
- Unstapled homeworks will not be accepted.
- Write your name, netID, and section on each homework.
- Homework are to be turned in to homework boxes on 3rd floor of ECEB. Please put your homework into the appropriate box for the section you are registered in:
  - Section X - 12:00 PM (Xu Chen): Box 35
  - Section E - 1:00 PM (Lynford Goddard): Box 36
  - Section F - 2:00 PM (Yang Zhao): Box 37
- Each student must submit individual solutions for each homework. You may discuss homework problems with other students registered in the course, but you may not copy their solutions. If you use any source outside of class materials that we've provided, you must cite every source that you used.
- Use of homework solutions from past semesters is not allowed and is considered cheating. Copying homework solutions from another student is considered cheating.
- Penalties for cheating on homework: 50% reduction in homework average on first offense, 100% reduction in homework average and report to college on second offense.

Reading Assignment: Kudeki: Lectures 1-3
Recommended Reading: Staelin: 1.1-1.3, 5.1, Appendix C

0. Register your iClicker, iClicker plus, or iClicker 2 by logging into Compass2G (https://compass2g.illinois.edu) and selecting "iClicker Registration" from the tab on the left. Please write down your iClicker Remote ID.

1. Consider the vectors in 3-D Cartesian coordinates:

$$\mathbf{A} = \hat{x} + 3\hat{y} - 2\hat{z},$$

$$\mathbf{B} = \hat{x} + \hat{y} - \hat{z},$$

$$\mathbf{C} = -\hat{x} + 3\hat{y} + 2\hat{z},$$

where $\hat{x} \equiv (1,0,0)$, $\hat{y} \equiv (0,1,0)$, and $\hat{z} \equiv (0,0,1)$ constitute an orthogonal set of unit vectors directed along the principal axes of a *right-handed* Cartesian coordinate system. Vectors can also be represented in component form — e.g., $\mathbf{A} = (1,3,-2)$, which is equivalent to $\hat{x} + 3\hat{y} - 2\hat{z}$.
Determine:

   a) The vector $\mathbf{A} - \mathbf{B} + 3\mathbf{C}$.
   b) The vector *magnitude* $|\mathbf{A} - \mathbf{B} + 3\mathbf{C}|$.
   c) The unit vector $\hat{u}$ along vector $\mathbf{A} + 2\mathbf{B} - \mathbf{C}$.
   d) The *dot products* $\mathbf{A} \cdot \mathbf{B}$ and $\mathbf{B} \cdot \mathbf{A}$.
   e) The *cross products* $\mathbf{B} \times \mathbf{C}$ and $\mathbf{C} \times \mathbf{B}$.
   f) Two of the vectors $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ are orthogonal. Specify which two and explain why.

2. Charges $Q_1 = -8\pi\epsilon_0\,\mathrm{C}$ and $Q_2 = \frac{-Q_1}{2}$ are located at points P_1 and P_2 having the position vectors $\mathbf{r}_1 = -\hat{z} = (0,0,-1)$ m and $\mathbf{r}_2 = \hat{z} = (0,0,1)$ m, respectively.

   a) Determine the electric field vector $\mathbf{E}$ at points P_3, P_4, and P_5 having the position vectors $\mathbf{r}_3 = (0,0,0)$ m, $\mathbf{r}_4 = 2\hat{z} = (0,0,2)$ m, and $\mathbf{r}_5 = -\hat{y} = (0,-1,0)$ m, respectively.
   b) Make sketches (3D perspective plots) showing the charge locations and the resulting electric field vectors — drawn coming out of points P_3, P_4, and P_5 respectively — in each case.

3. A particle with charge $q = 1$ C passing through the origin $\mathbf{r} = (x,y,z) = 0$ of the lab frame is observed to accelerate with forces

$$\mathbf{F}_1 = 2\hat{x} - \hat{y}, \quad \mathbf{F}_2 = 2\hat{x} - 3\hat{y}, \quad \mathbf{F}_3 = 3\hat{x} - 2\hat{y} - \hat{z} \quad \mathrm{N}$$

when the velocity of the particle is

$$\mathbf{v}_1 = 0, \quad \mathbf{v}_2 = -2\hat{z}, \quad \mathbf{v}_3 = \hat{x} + \hat{y} \quad \frac{\mathrm{m}}{\mathrm{s}},$$

in turns. Use the Lorentz force equation $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ to determine the fields $\mathbf{E}$ and $\mathbf{B}$ at the origin.
Hint: Assume the $\mathbf{B}$ and $\mathbf{E}$ fields are the same for all three measurements, then consider a $\mathbf{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$ and solve the three vector equations obtained from $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ for the unknowns B_x, B_y, B_z, and $\mathbf{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$.

4. Let $\mathbf{J} = x^2y(\hat{x}+\hat{y}+\hat{z})\,\frac{A}{m^2}$ denote the electrical *current density* field — i.e., current flux per unit area — in a region of space represented in Cartesian coordinates. A current density of $\mathbf{J} = x^2y(\hat{x}+\hat{y}+\hat{z})\,\frac{A}{m^2}$ implies the flow of electrical current in direction $\frac{\mathbf{J}}{|\mathbf{J}|} = \frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$ with a magnitude of $|\mathbf{J}| = x^2y\sqrt{3}$ amperes (A) per unit area.

   a) Calculate the total *current flux* $\oint_S \mathbf{J}\cdot d\mathbf{S}$ out of a closed surface S enclosing a cubic volume $V = 8$ m^3 with vertices at $(x, y, z) = (0, 0, 0)$ and $(2, 2, 2)$ m. Be sure to clearly indicate the units of your solution.
   Hint: Surface S of cube V consists of six surfaces of square shapes having equal areas $S_i = 4$ m^2, $i = 1, 2, \cdots, 6$. The flux $\oint_S \mathbf{J}\cdot d\mathbf{S}$ is therefore the sum of six *surface integrals* $\int_{S_i} \mathbf{J}\cdot d\mathbf{S}$ taken over surfaces S_i, where the infinitesimal area vectors $d\mathbf{S}$ are, in turn, $\pm\hat{z}dxdy$, $\pm\hat{x}dydz$, and $\pm\hat{y}dzdx$ — by convention $d\mathbf{S}_i$ are taken as vectors pointing *away* from volume V (at each subsurface S_i) in flux calculations.

   b) Given the result of part (a), do you expect the total amount of electrical charge Q_V contained in volume V to increase or decrease? In answering this question assume that electric charge is *conserved* (as in real life). Remember that electrical current represents electrical charges in motion.

   c) Given the result of part (b), do you expect the electric field flux $\oint_S \mathbf{E}\cdot d\mathbf{S}$ out of the same enclosed surface S to increase or decrease? Explain in terms of Gauss's law.

5. A charge-neutral region of space contains stationary electrons (having charge $q = -e$, mass m_e, and number density $N_e = 10^6$ cm^{-3}) and protons (having charge $q = +e$, mass m_p, and number density $N_p = N_e$). At time $t = 0$, a static and uniform electric field $\mathbf{E} = E_0\hat{z}\,\frac{V}{m}$ is switched on, which immediately generates an associated Lorentz force on the charges.

   a) Treating the charged particles as "test charges" (such that the Coulomb electric fields generated by the charges themselves can be neglected), what is the magnitude and direction (i.e., the unit vector) of the Lorentz acceleration experienced by a single proton and electron, respectively?

   b) For time $t > 0$, the electrons move under the influence of the Lorentz force with a velocity $v_e(t)$ while the protons move with a velocity $v_p(t)$. If $E_0 = 2\,\frac{mV}{m}$, where would a single test proton and electron be located (in 3D space) after 1 second elapse, assuming both charged particles were located at the origin, $\mathbf{r}_0 = (0, 0, 0)$, at $t = 0$? Express your answer in kilometers.

   c) The moving charges constitute a current. What is the total volumetric current density $\mathbf{J}$ (in units of $\frac{A}{m^2}$) carried by both the ions and electrons in the region at $t > 0$? **Hint:** Start by expressing the current density $\mathbf{J}$ in terms of the charge density $\rho = qN\,\frac{C}{m^3}$.

6. **Bonus Problem:** Some homeworks will contain an *optional* bonus problem, which can help you increase your homework average for the semester up to a maximum of 100%.
   Divergence theorem is a concept from MATH 241 that is heavily utilized in this course. Use the fact that
   $$\nabla\cdot\mathbf{A} = \lim_{\Delta V\to 0}\frac{1}{\Delta V}\left[\oint_S \mathbf{A}\cdot d\mathbf{S}\right]$$
   to derive (either mathematically, in words, or graphically) the definition for divergence in Cartesian coordinates:
   $$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$
   and the divergence theorem:
   $$\oint_S \mathbf{A}\cdot d\mathbf{S} = \int_V \nabla\cdot\mathbf{A}\,dV.$$