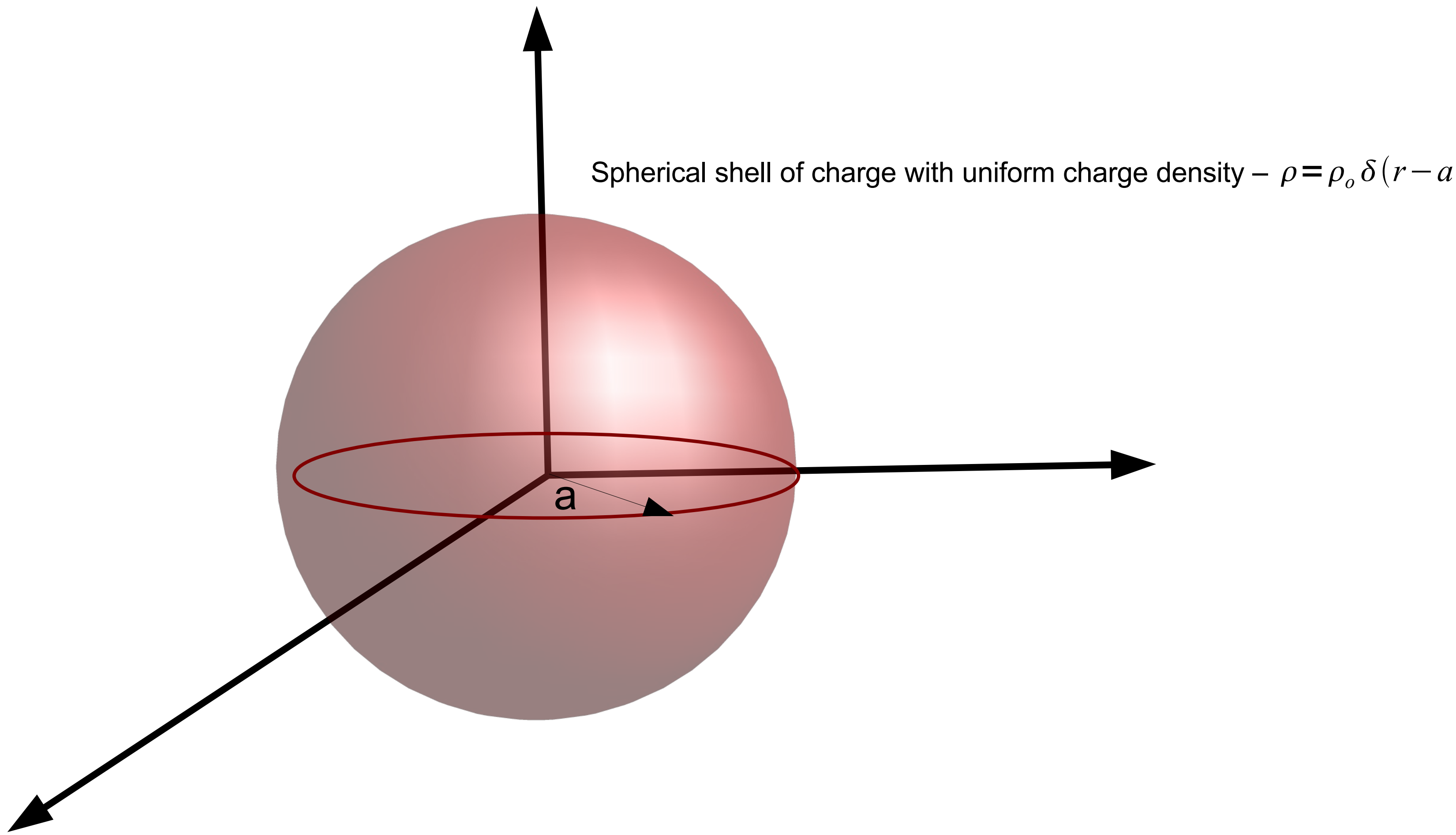
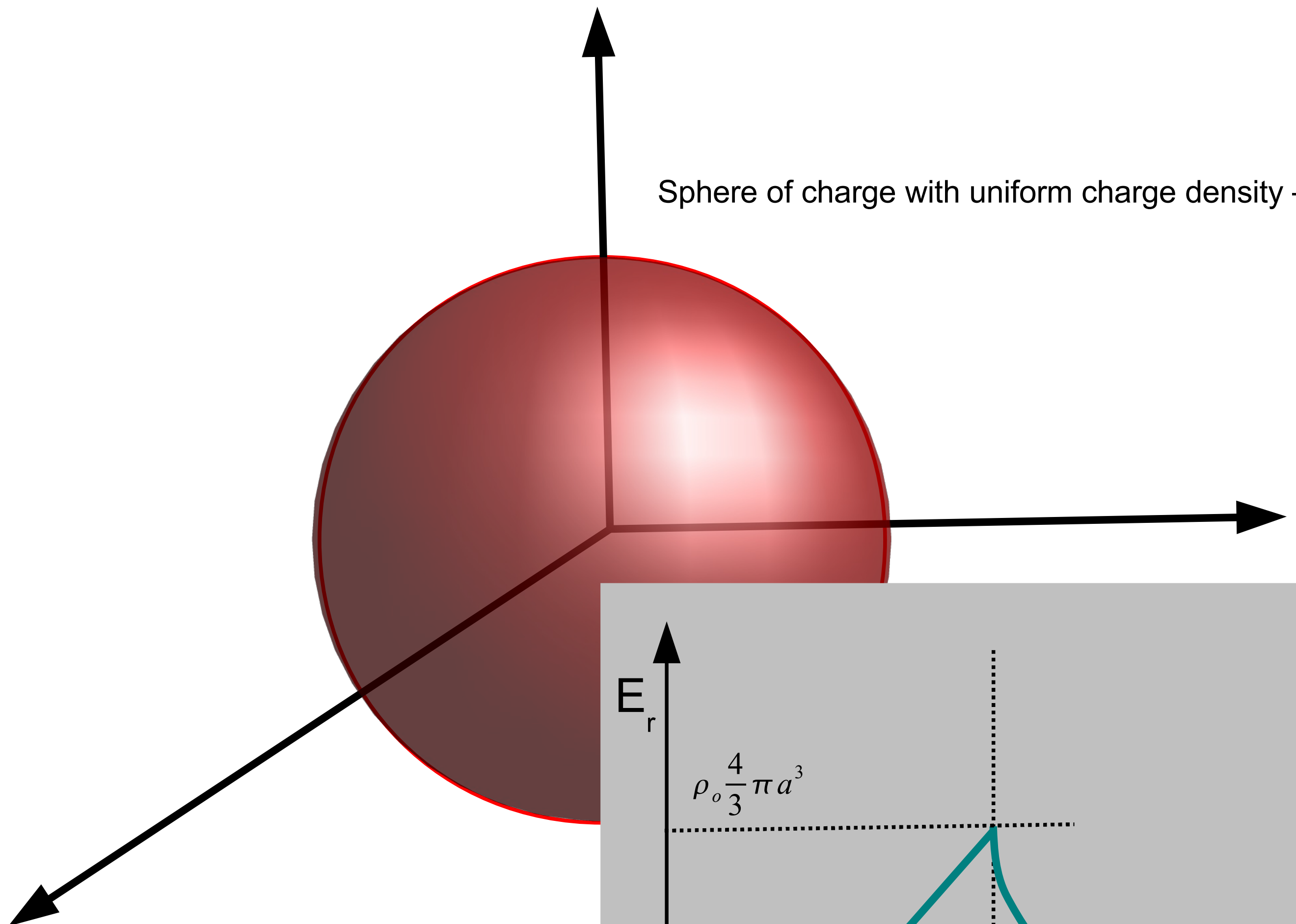
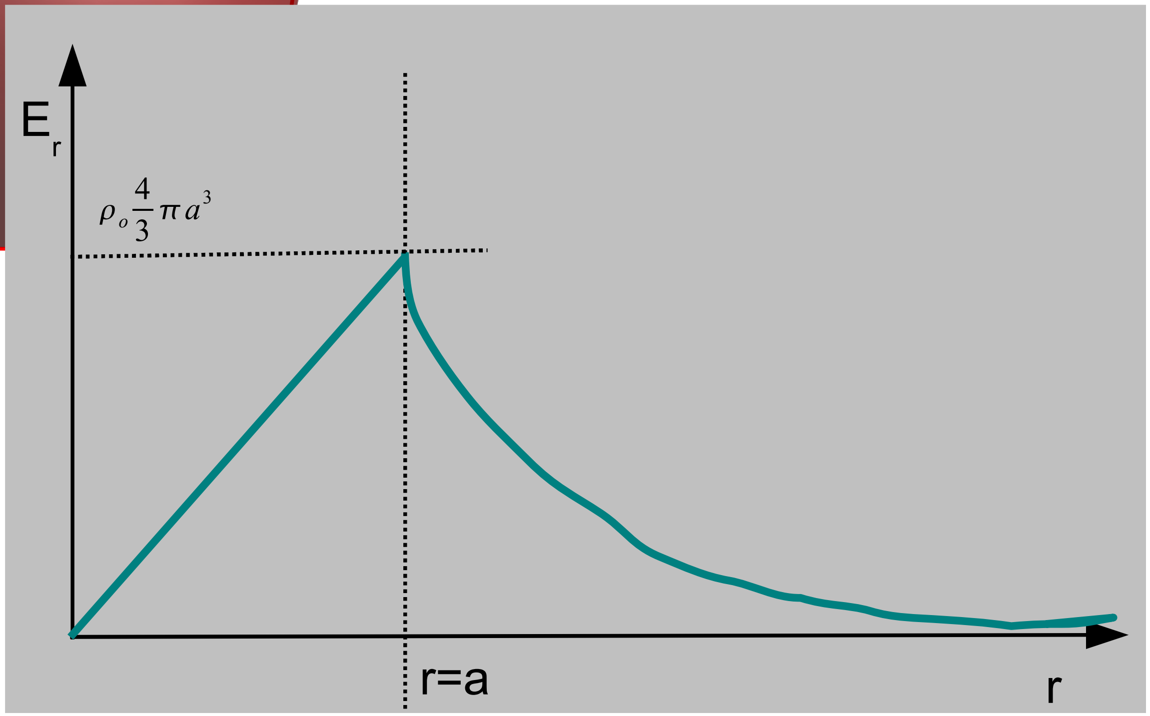
Find the electric field and the voltage everywhere.



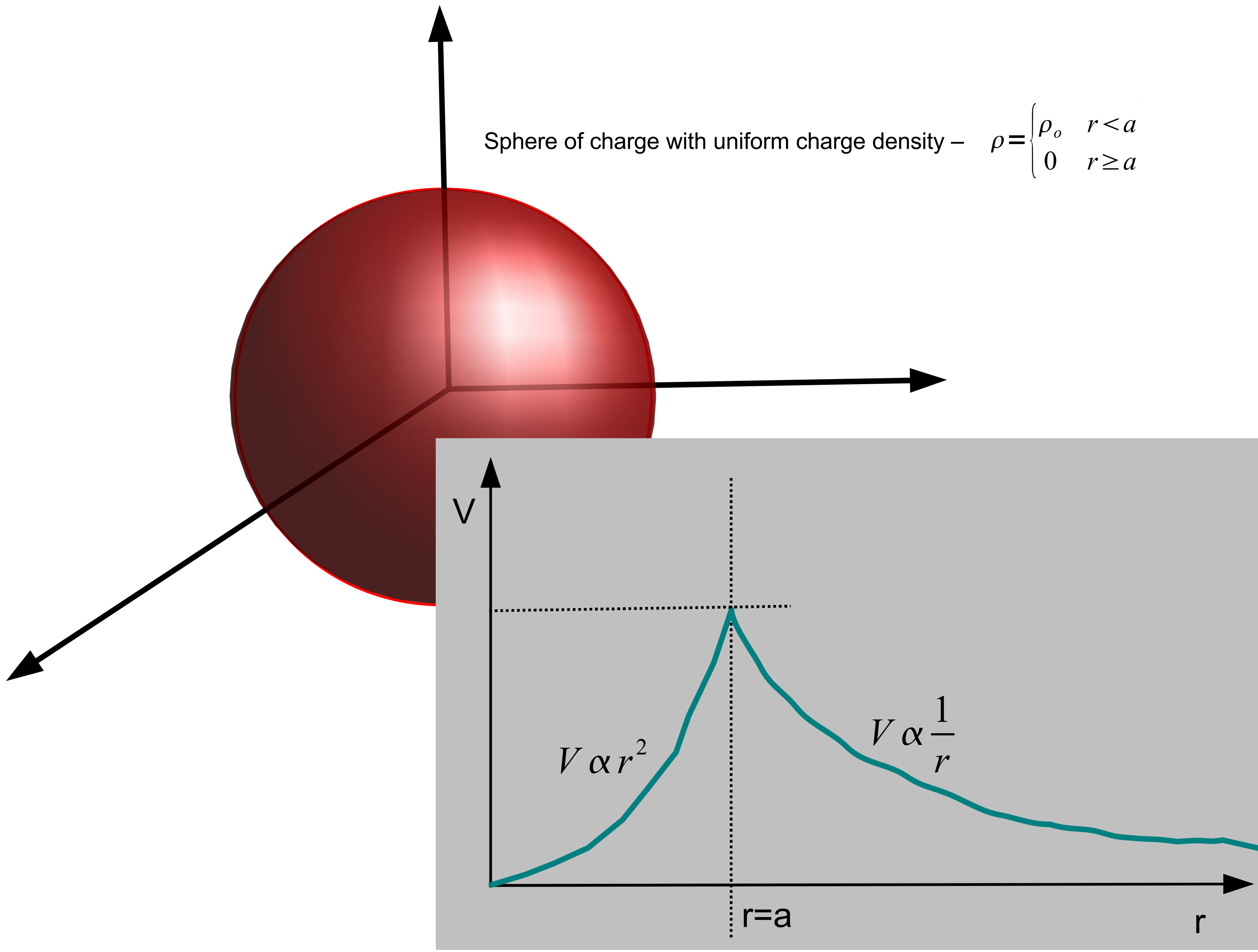
Find the electric field and the voltage everywhere.

Sphere of charge with uniform charge density – $\rho = \begin{cases} \rho_o & r < a \\ 0 & r \geq a \end{cases}$





Find the electric field and the voltage everywhere.



Find the electric field and the voltage everywhere.

From Gauss' law in integral form it is easy to derive the electric field.

$$\oint \vec{E}\cdot d\vec{S} = \frac{Q}{\epsilon_o}$$

Combining the results...

$$\oint \vec{E}\cdot d\vec{S} = 4\pi r^2 E_r(r)$$

and...

In the region r<a:

$$\frac{Q}{\epsilon_o} = \frac{\rho_o}{\epsilon_o}\frac{4}{3}\pi r^3$$

In the region r>a:

$$\frac{Q}{\epsilon_o} = \frac{\rho_o}{\epsilon_o}\frac{4}{3}\pi a^3$$

$$E_r(r) = \left\{ \begin{matrix} \dfrac{\rho_o r}{3\epsilon_o} & r<a \\ \dfrac{\rho_o a^3}{3\epsilon_o r^2} & r\geq a \end{matrix} \right\}$$

Find the electric field and the voltage everywhere.

From Gauss' law in differential form it is easy to derive the electric field.

$$\nabla \cdot \vec{E} = \frac{\rho_o}{\epsilon_o}$$

By symmetry we assume $\vec{E} = E_r(r)\hat{r} \cdot$

**r<a**

$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 E_r = \rho_o$$

$$E_r = \frac{\rho_o r}{3\epsilon_o} + \frac{A_I}{r^2}$$

**r>=a**

$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 E_r = 0$$

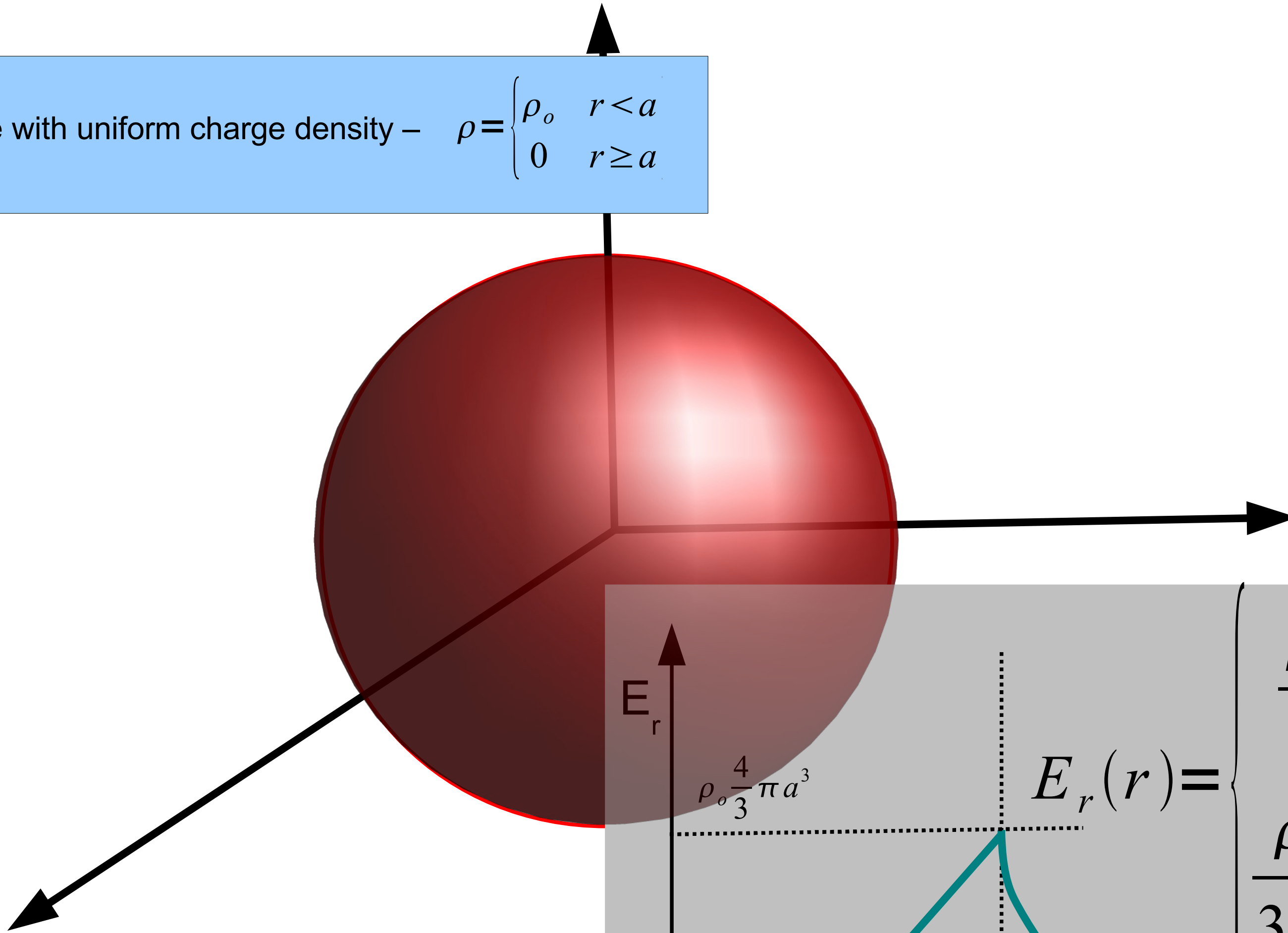
$$E_r = \frac{A_{II}}{r^2}$$

Find A and A by applying boundary conditions

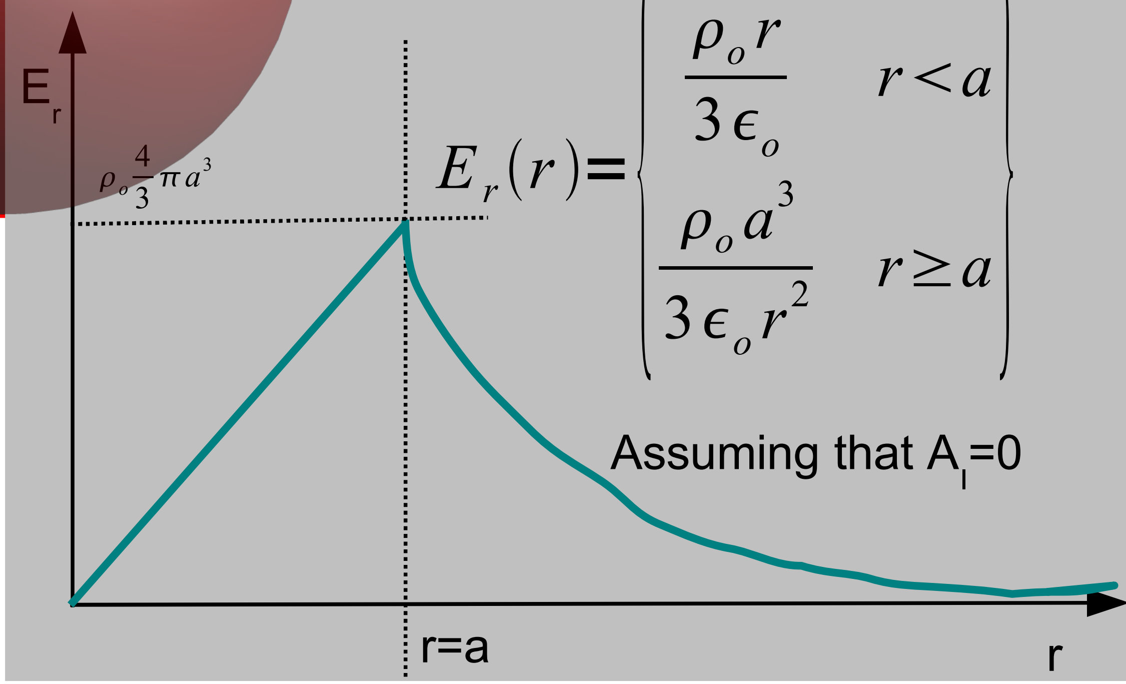
$$\frac{\rho_o a}{3\epsilon_o} + \frac{A_I}{a^2} = \frac{A_{II}}{a^2} \Rightarrow A_{II} = \frac{\rho_o a^3}{3\epsilon_o} + A_I$$

Find the electric field and the voltage everywhere.

Sphere of charge with uniform charge density – $\rho = \begin{cases} \rho_o & r < a \\ 0 & r \geq a \end{cases}$



$$E_r(r) = \begin{cases} \dfrac{\rho_o r}{3\epsilon_o} & r < a \\[2ex] \dfrac{\rho_o a^3}{3\epsilon_o r^2} & r \geq a \end{cases}$$




Assuming that $A_I=0$

Now let's find the voltage everywhere:

$$E_r(r) = \begin{cases} \dfrac{\rho_o r}{3\epsilon_o} & r < a \\ \dfrac{\rho_o a^3}{3\epsilon_o r^2} & r \geq a \end{cases}$$

Differential form: $\vec{E} = -\nabla V$

$$E_r(r) = -\frac{\partial V}{\partial r}$$

$$-\frac{\partial V}{\partial r} = \begin{cases} \dfrac{\rho_o r}{3\epsilon_o} & r < a \\ \dfrac{\rho_o a^3}{3\epsilon_o r^2} & r \geq a \end{cases}$$

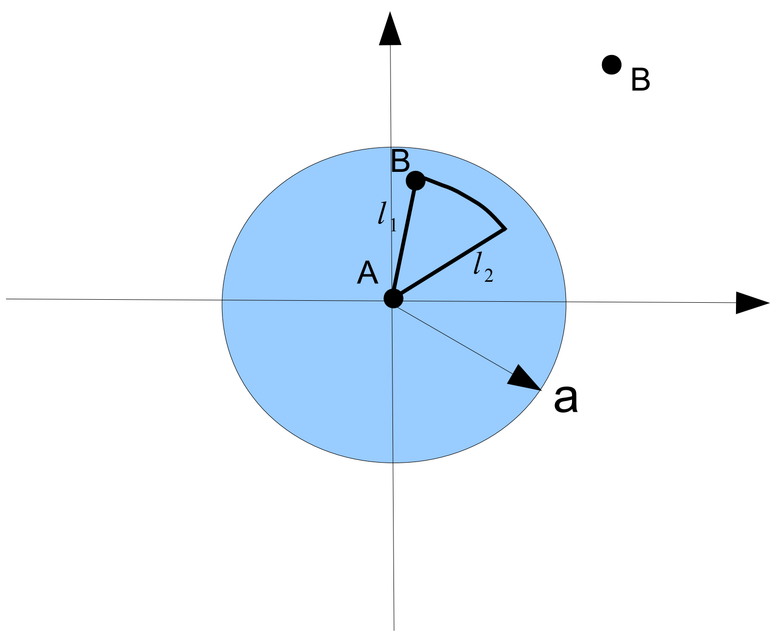
$$V = \begin{cases} -\dfrac{\rho_o r^2}{6\epsilon_o} + A_I & r < a \\ \dfrac{\rho_o a^3}{3\epsilon_o r} + A_{II} & r \geq a \end{cases}$$

$$V = \begin{Bmatrix} -\dfrac{\rho_o r^2}{6\epsilon_o} + A_I & r < a \\ \dfrac{\rho_o a^3}{3\epsilon_o r} + A_{II} & r \geq a \end{Bmatrix}$$

Now let's find the voltage everywhere:

$$E_r(r) = \begin{Bmatrix} \dfrac{\rho_o r}{3\epsilon_o} & r<a \\ \dfrac{\rho_o a^3}{3\epsilon_o r^2} & r \geq a \end{Bmatrix}$$

Differential form: $V = -\int_A^B \vec{E} \cdot d\vec{l} = V(A) - V(B)$



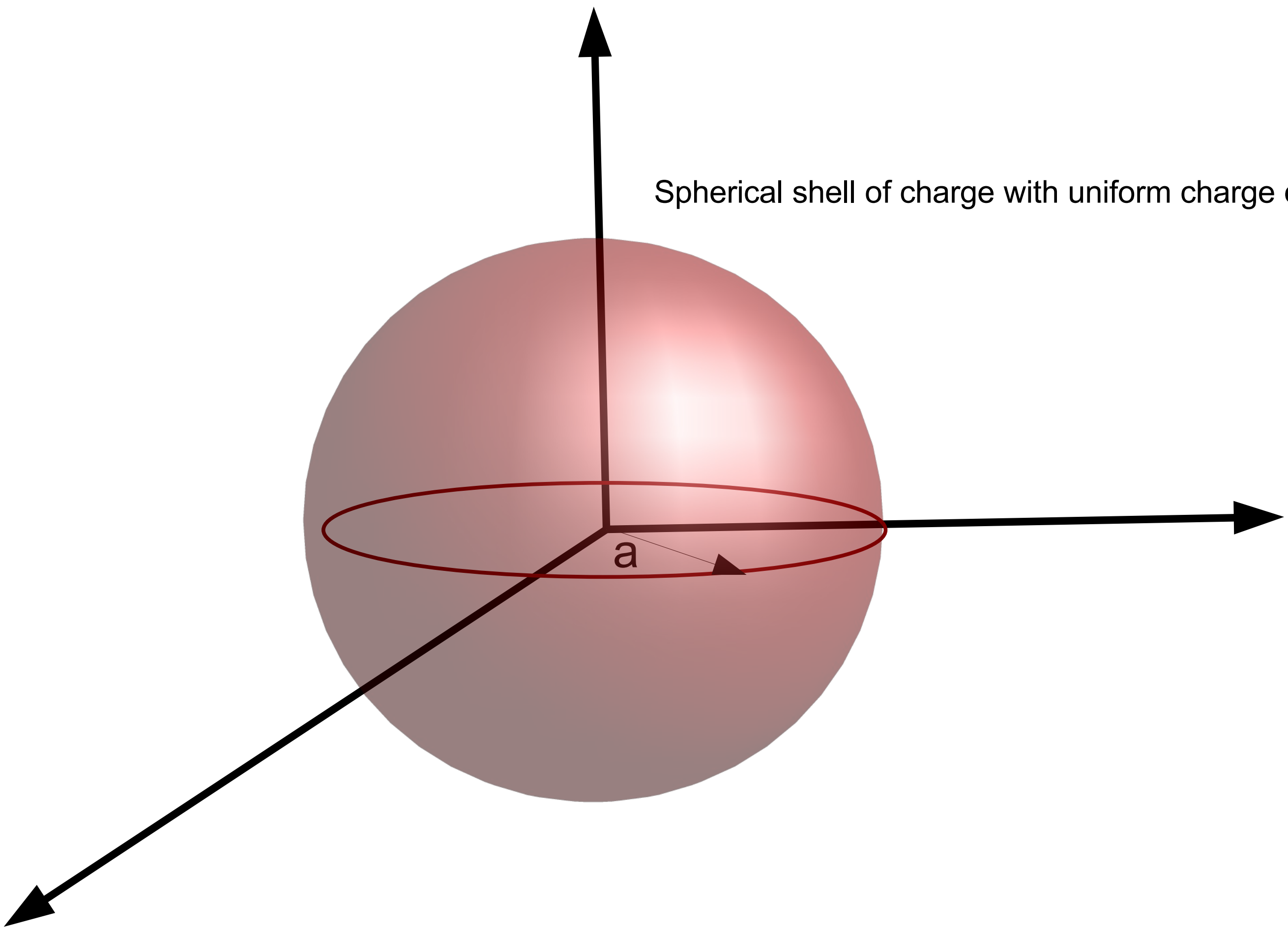

r<a

$$= -\int_0^r \frac{\rho_o r'}{3\epsilon_o} dr' = \frac{-\rho_o r^2}{6\epsilon_o}$$

r>=a

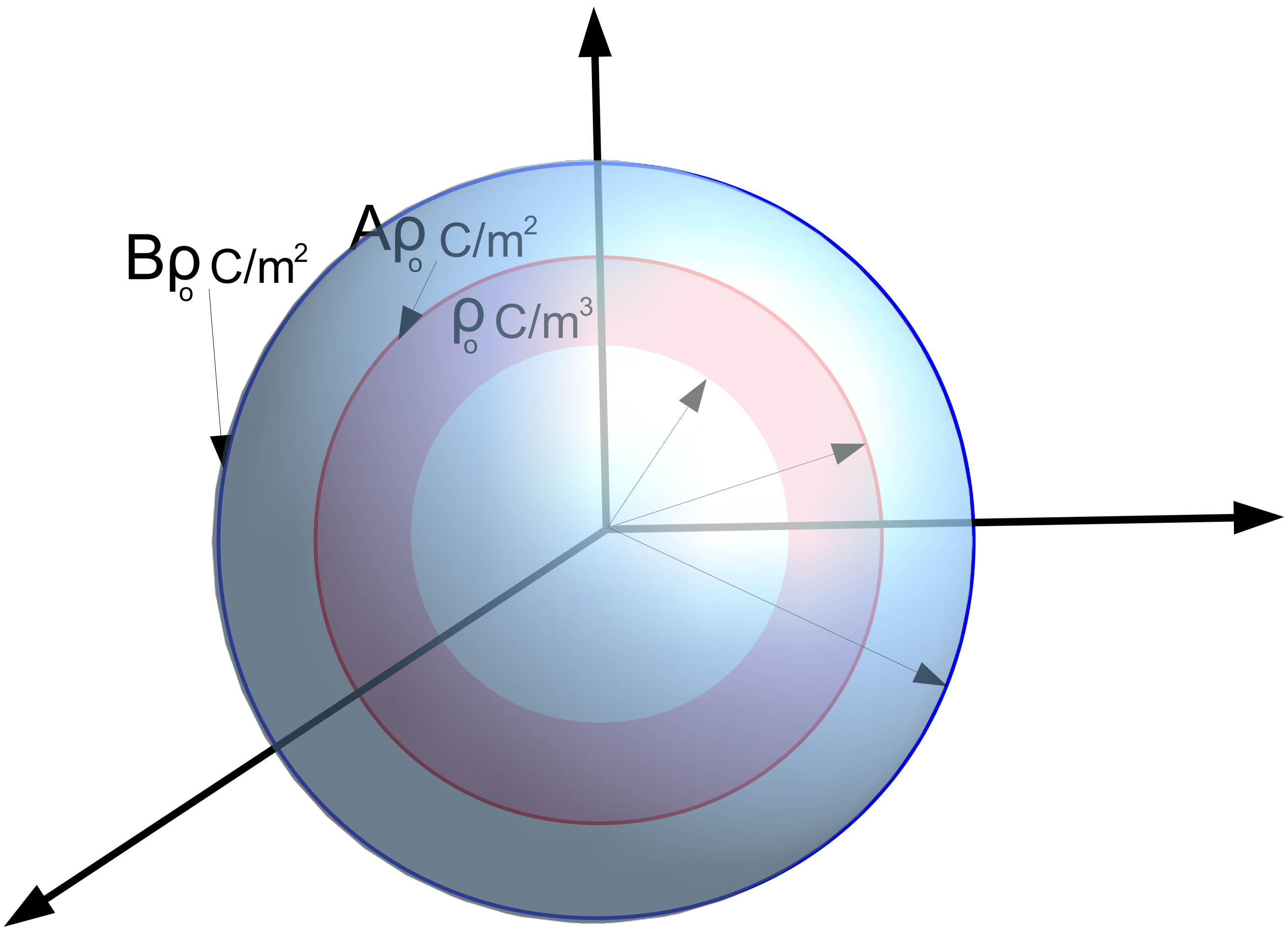
$$= -\int_0^r \frac{\rho_o a^3}{3\epsilon_o r^2} dr' = \frac{\rho_o a^3}{3\epsilon_o r}$$

Find the electric field and the voltage everywhere.




Spherical shell of charge with uniform charge density – $\rho = \rho_o \delta(r-a)$

Find the electric field and the voltage everywhere.



$$\vec{E} = \frac{Q}{4\pi\epsilon_o r^2}\hat{r}$$

Coulombs law

$$\int_s \vec{E}\cdot d\vec{S} = \frac{Q}{\epsilon_o}$$

$$\oint_c \vec{E}\cdot d\vec{l} = 0$$

Maxwell's equations
Integral form

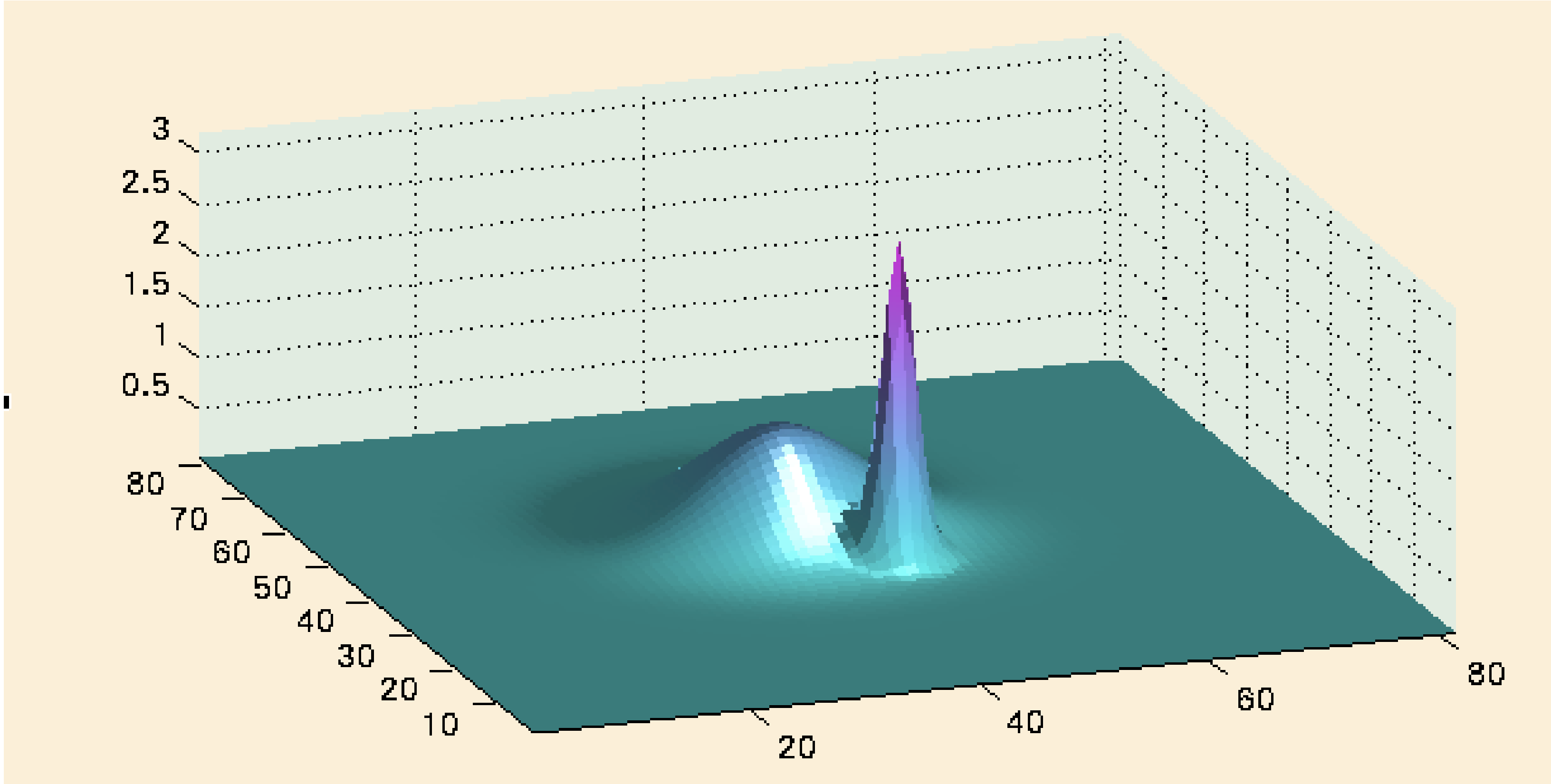
$$\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_o}$$

$$\nabla\times\vec{E} = 0$$

Maxwell's equations
Differential form

$$\nabla^2 V = \frac{-\rho}{\epsilon_o}$$

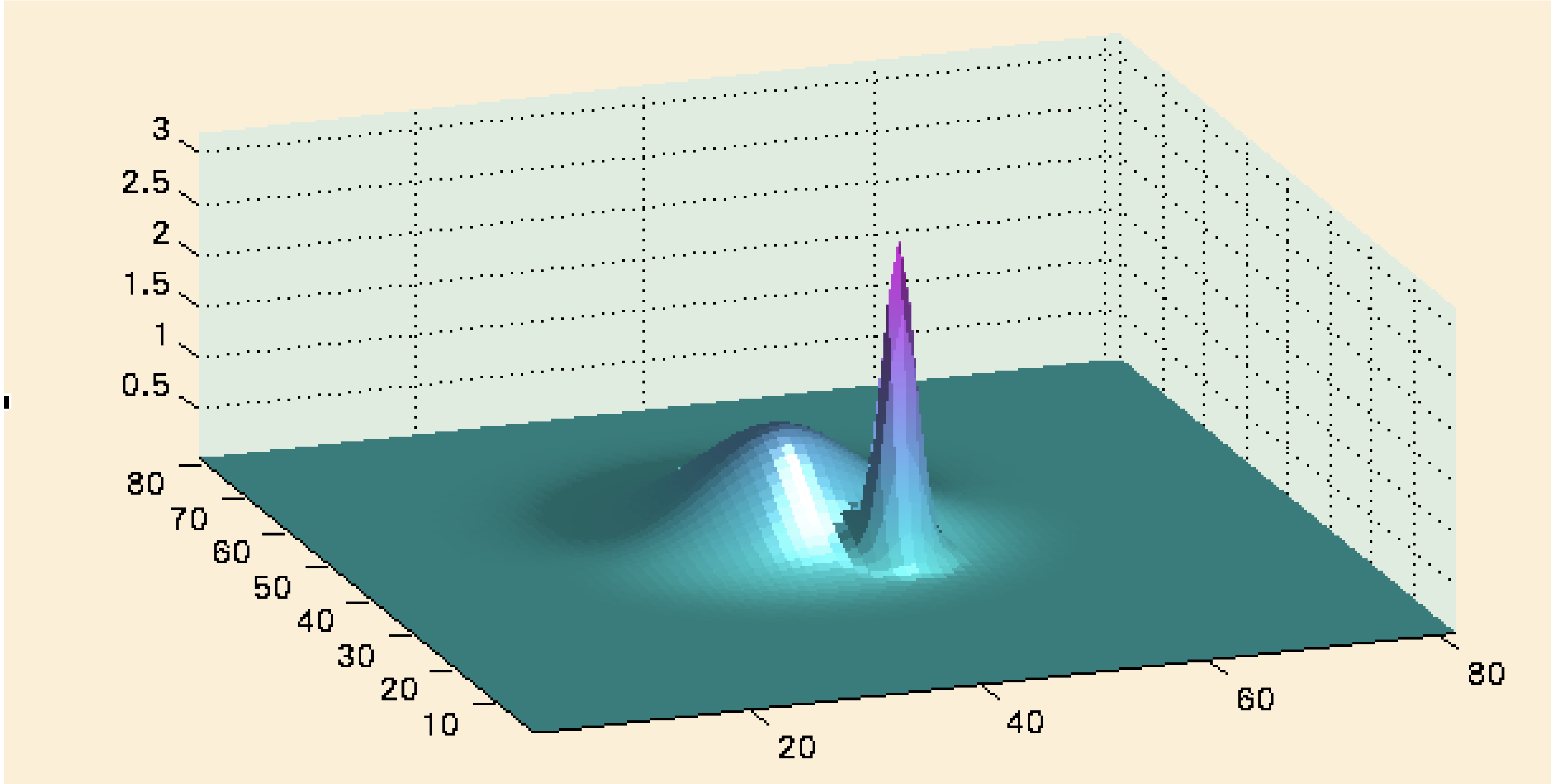
Maxwell's equations
Differential form
potential function




$$V(x,y)=3e^{-(x-2)^2}e^{-(y+3)^2}+e^{-(x+1)^2}e^{-(y-3)^2/25}$$

Find the electric field and the charge distribution that generated the field.

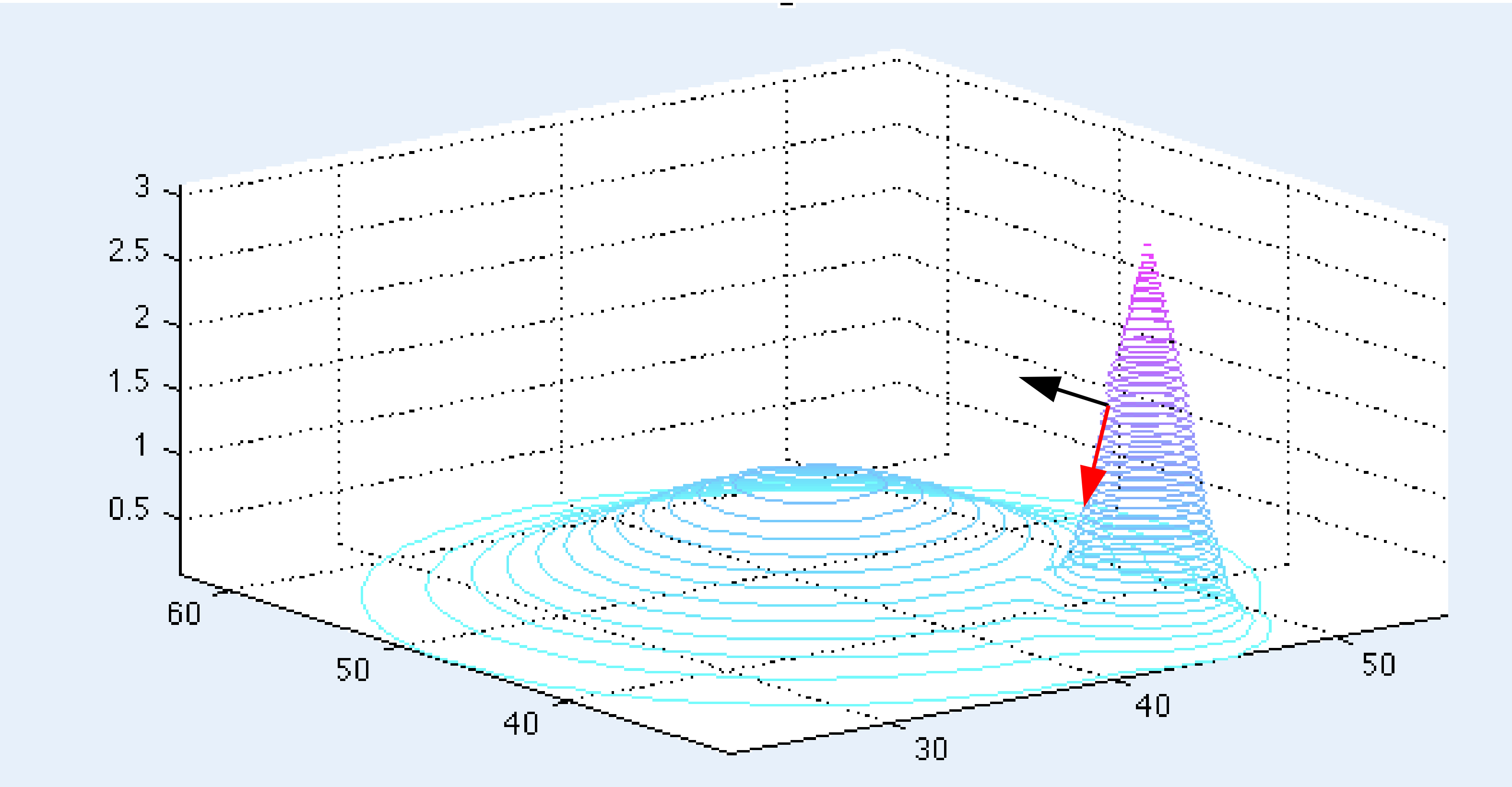
$$\vec{E}=-\nabla V$$

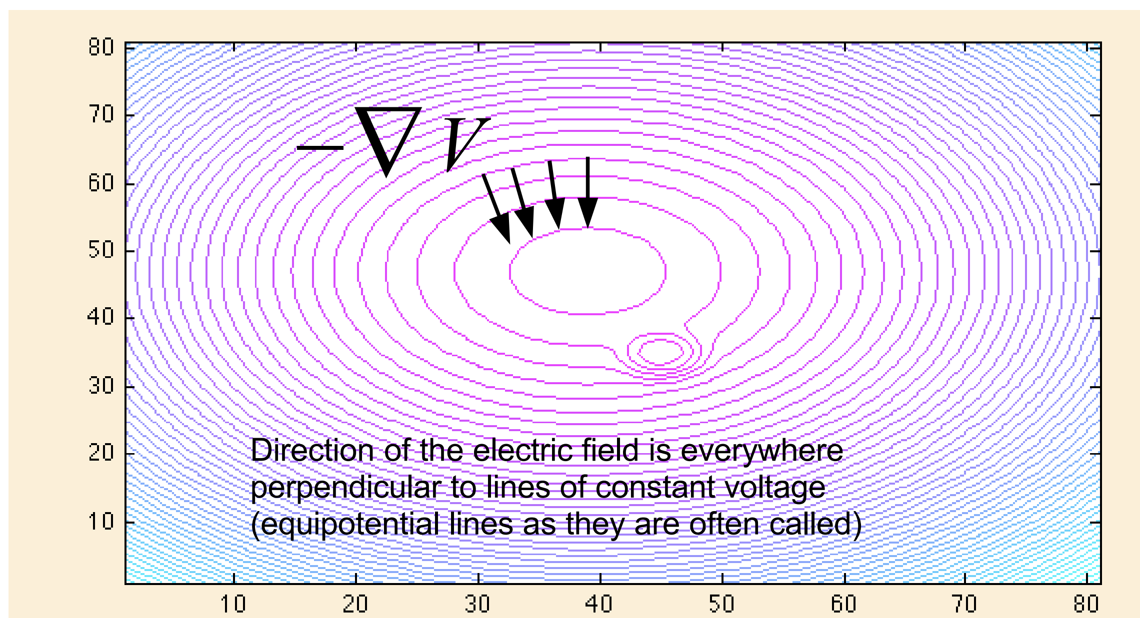



$$V(x,y) = 3e^{-(x-2)^2}e^{-(y+3)^2} + e^{-(x+1)^2}e^{-(y-3)^2/25}$$

Find the negative of the gradient of V to retrieve the electric field magnitude ----

$$\vec{E} = \left(-6(x-2)e^{-(x-2)^2}e^{-(y+3)^2} - 2(x+1)e^{-(x+1)^2}e^{-(y-3)^2/25}\right)\hat{x}$$
$$+\left(-6(y+3)e^{-(x-2)^2}e^{-(y+3)^2} - 2(y-3)e^{-(x+1)^2}e^{-(y-3)^2/25}\right)\hat{y}$$


3
2.5
2
1.5
1
0.5
60
50
40
30
40
50


80
70
60
50
40
30
20
10
10
20
30
40
50
60
70
80
$-\nabla V$
Direction of the electric field is everywhere
perpendicular to lines of constant voltage
(equipotential lines as they are often called)

$$V(x, y, z) = x + 2yz^2 + x(y+3)$$