University of Illinois Summer 2018

ECE 313: Exam II

Wednesday, July 11, 2018
10 - 10.50 a.m.
1013 ECEB

Name: (in BLOCK CAPITALS) ____________________________________

NetID: ____________________________________

Signature: ____________________________________

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of **4** problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the blank page at the end of the exam.

Grading

1. 27 points ____________
2. 23 points ____________
3. 20 points ____________
4. 30 points ____________

Total (100 points) ____________

1. **[27 points]** Suppose that you have three biased coins and a fair die. Coin C_1 has $P\{heads\} = 1/3$, while coins C_2 and C_3 have $P\{heads\} = 1/4$. Flip coin C_1 10 times and let X_1 be the total number of heads that show. Flip coin C_2 5 times and let X_2 be the total number of heads that show. Let $Y = X_1 + X_2$.

   (a) Determine $E[X_1]$ and $E[Y]$.

   (b) Obtain $P\{X_1 = 1\}$ and $P\{Y = 1\}$.

   (c) Roll the die and let N be the number showing. Then flip coin C_3 N times. Let Z be the number of heads showing. Find $E[Z]$.

2. **[23 points]** Suppose a fair die is rolled. Let X be the number of times the die is rolled until an even number shows. Let N be that first even number that shows, and let Y be the number of additional times the die is rolled until N shows again.

   (a) Determine the pmf of X, and its mean μ_X.

   (b) Determine the pmf of Y, and its mean μ_Y.

   (c) Determine $E[X - Y]$.

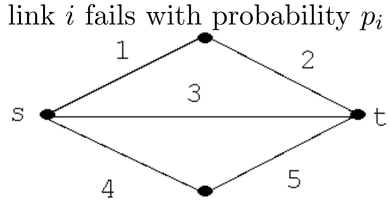
3. **[20 points]** The two parts of this problem are unrelated.

(a) Le X be a random variable with pmf

$$p_X(k) = \begin{cases} \frac{1}{50}p & k \in \{1, 3, 5, \ldots, 99\} \\ \frac{1}{50}(1-p) & k \in \{2, 4, 6, \ldots, 100\}, \end{cases}$$

where p is unknown. The experiment is performed once and it is observed that $X = 16$. Determine the maximum likelihood estimate $\hat{p}_{ML}$.

(b) Determine the probability of network outage in the $s - t$ network below, assuming that link i fails with probability p_i independently of other links.



4. **[30 points]** Consider a binary hypothesis testing problem with the following likelihood matrix:

	$X=1$	$X=2$	$X=3$	$X=4$	$X=5$
H_0	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{4}{12}$	x	$\frac{5}{12}$
H_1	$\frac{2}{12}$	$\frac{1}{12}$	y	$\frac{3}{12}$	$\frac{4}{12}$

(a) Determine the values of x and y.

(b) Determine the ML rule.

(c) Obtain a decision rule such that $p_{miss} = \frac{1}{12}$.

(d) If the MAP rule is used, for which value(s) of prior π_1 would H_1 always be declared?

You may use this sheet for additional calculations but do not separate this sheet from the rest of the exam.

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