ECE 313: Exam II

Thursday, July 21, 2016
8.45-10 p.m.
ECEB 1013

Name: (in BLOCK CAPITALS) _____

NetID: _____

Signature: _____

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheets of notes is permitted: both sides may be used. No electronic equipment (cell phones, etc.) allowed.

The exam consists of ?? problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading

1. 18 points _____

2. 28 points _____

3. 18 points _____

4. 20 points _____

5. 16 points _____

Total (100 points) _____

1. **[18 points]** Let A be a real-valued constant and consider the function

$$F_X(u) = \begin{cases} 0 & u < -2 \\ \frac{1}{2}u + 1 & -2 \le u \le -1 \\ \frac{1}{2} & -1 < u < 1 \\ 1 - Ae^{-u} & u \ge 1 \end{cases}.$$

(a) Find all values of the constant A that make this a valid CDF.

(b) Obtain $P\{X \le -1\}$.

(c) Obtain $P\{X < 1\}$.

(d) Obtain $P\{X = -1\}$.

(e) Obtain $P\{|X| < 1\}$.

2. **[28 points]** Let X be an exponential random variable with parameter 2.

(a) Determine $P\{X > 5\}$.

(b) Determine $P\{X < 8 | X > 5\}$.

(c) Determine $E[X^2 | X > 5]$.

(d) Recall that $X \sim Exp(2)$. Determine $E[e^{-2X}]$.

(e) Let $Z = e^{-2X} + 2$. Obtain the pdf of Z, $f_Z(v)$, for all v.

(f) Determine $E[Z]$.

3. **[18 points]** Let X be Gaussian with mean -1 and variance 16.

(a) Express $P\{X^3 \leq -8\}$ in terms of the Φ function.

(b) Let $Y = \frac{1}{2}X + \frac{3}{2}$. Sketch the pdf of Y, $f_y(v)$, clearly marking important points.

(c) Express $P\{Y \geq \frac{1}{2}\}$ in terms of the Φ function.

4. [**20 points**] Suppose the number of fish detected by a remote underwater sensor follows a Poisson process with rate 5 fish per minute.

    (a) What is the probability that exactly two fish are detected in a three minute period?

    (b) What is the probability that exactly two fish are detected each minute in three consecutive minutes?

    (c) What is the probability that exactly two fish are detected the first minute given that exactly three fish are detected in the first four minutes?

    (d) What is the expected value of the number of fish detected in the first minute given that exactly three fish are detected in the first four minutes?

5. [**16 points**] Suppose under hypothesis H_1, X has pdf $f_1(u) = e^{-2|u|}$ for all u, but under hypothesis H_0, X is exponentially distributed with parameter one. Let $\pi_0 = \frac{2}{3}$.

(a) Obtain the MAP decision rule.

(b) Obtain $p_{false\ alarm}$ for the MAP rule.

(c) Obtain p_{miss} for the MAP rule.