University of Illinois Summer 2016

ECE 313: Exam I

Thursday, July 7, 2016
8.45 - 10 p.m.
1013 ECEB

Name: (in BLOCK CAPITALS) ______________________________

NetID: ______________________________

Signature: ______________________________

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of **???** problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading	
1. 16 points	________
2. 12 points	________
3. 20 points	________
4. 18 points	________
5. 16 points	________
6. 18 points	________
Total (100 points)	________

1. **[16 points]** Consider three events A, B, and E such that: $P(A) = 0.3$, $P(AB) = 3P(ABE)$, $P(A \cup E) = 0.5$, $P(AB^c) = 0.03$, $P(BE) = 0.1$ and $P(A^cB^cE^c) = 0.48$.

   (a) Obtain $P(ABE)$.

   (b) Obtain $P(A^cBE^c)$.

   (c) Are events AB and AE independent?

2. **[12 points]** The two parts of this problem are unrelated.

   (a) Consider a poker hand. Obtain $P\{\text{full house}\}$, where a full house consists of a three of a kind and a pair, that is three cards with the same rank and also two cards with the same rank but different than the rank of the other three cards.

   (b) Suppose that you have n distinct pairs of mittens. You put all inside a black bag and mix them, then you take out four mittens at random. How many ways are there to get at least one left mitten and two right mittens?

3. **[20 points]** Dilbert, Wally and Alice are tossing fair coins simultaneously but independently of each other. So they independently toss them at the same time and look at the three coins laying on the floor. Let X be the first time *at least* one of them gets a head on their simultaneous tosses.

   (a) Find the probability that X is strictly larger than 2.

   (b) Calculate $E[X]$.

   (c) Find $E[X^2]$.

   (d) Calculate $E[X^2|X > 2]$.

4. **[18 points]** A student is taking a multiple choice test with 10 questions. He did not study, so he answers each question by randomly choosing among the 5 possible answers, independently of any other questions. Let $X_i = 1$ if the student answers the i-th question correctly, and $X_i = 0$ if he doesn't. Let X be the total number of questions he answers correctly.

   (a) Obtain $P\{X = 3 | X_1 = 1, X_{10} = 0\}$.

   (b) Obtain $P\{X_1 = 1 | X_{10} = 0, X = 3\}$.

   (c) If the student gets 2 points for each correct answer, obtain the mean and variance of the number of points he will get.

5. **[16 points]** Let X be uniform on the set $\{n, n+1, \ldots, 2n\}$.

(a) Assume that $n = 3$. Obtain $E[X]$.

(b) Assume again that $n = 3$. Obtain $E\left[\frac{1}{X}\right]$.

(c) Assume now that that n is unknown, but after performing the experiment, it is observed that $X = 5$. Obtain the maximum likelihood estimate of n, denoted by $\hat{n}_{ML}$.

6. **[18 points]** Consider the binary hypothesis testing where X is uniform over the integers $\{1, 2, \ldots, 10\}$ under hypothesis H_0 and $Geometric(1/2)$ under hypothesis H_1.

(a) Derive the ML rule to decide on the underlying hypothesis.

(b) What is the probability of false alarm for the ML rule?

(c) What is the probability of missed detection for the ML rule?