University of Illinois Summer 2015

ECE 313: Exam II

Thursday, July 23, 2015
4:00 p.m. — 5:15 p.m.
1013 ECEB

Name: (in BLOCK CAPITALS) ____

NetID: ____

Signature: ____

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. No electronic equipment (cell phones, etc.) allowed.

The exam consists of six problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

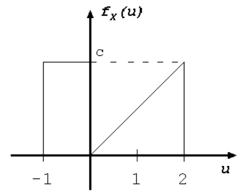
SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading

1. 22 points ____________
2. 14 points ____________
3. 25 points ____________
4. 14 points ____________
5. 25 points ____________

Total (100 points) ____________

1. **[22 points]** Let X have the probability density function (pdf) $f_X(u)$ plotted below.



(a) Obtain the constant c for $f_X(u)$ to be a valid pdf.

(b) Obtain the cumulative distribution function (CDF) of X, $F_X(u)$ for all u.

(c) Let $Y = |X|$, obtain the cumulative distribution function (CDF) of Y, $F_Y(v)$ for all v.

2. **[14 points]** Let $X \sim Uniform\left(0, \frac{1}{a}\right)$, that is, X has a uniform distribution on $\left(0, \frac{1}{a}\right)$.

(a) If a is unknown, but it is observed that $X = \frac{1}{4}$, obtain the maximum likelihood estimate $\hat{a}_{ML}$ of a.

(b) If $Y = 2X + 1$ and it is known that $Var(Y) = 12$, obtain a.

3. **[25 points]** Bikes arrive at a certain intersection according to a Poisson process with arrival rate $\lambda = 2$ bikes/hour. Let N_t denote the number of bikes that arrive up until time t. Obtain:

   (a) $P\{$it takes more than 2 hours after the first bike arrives until the second bike arrives$\}$.

   (b) $P\{N_3 - N_{1.5} = 2\}$.

   (c) $P\{N_3 = 8 | N_1 = 3\}$.

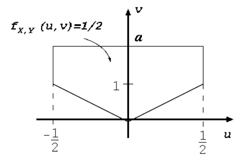
   (d) Suppose λ is unknown but it is known that $P\{N_1 = 3 | N_3 = 8\} = \binom{8}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^5$, obtain the possible values of λ.

4. **[14 points]** Let $X \sim N(\mu, \sigma^2)$, that is, X is a Gaussian random variable with mean μ and variance σ^2. It is known that $P\{X > 2\} = 0.5$ and that $P\{X < 0.14\} = 0.2676$.

(a) Obtain μ and σ^2.

(b) Find constants a and b such that $Y = aX + b$ is a Gaussian random variable with mean 1 and variance $\frac{1}{4}$, that is, for $Y \sim N\left(1, \frac{1}{4}\right)$.

5. **[25 points]** Let X and Y be jointly continuous random variables with joint probability density function (pdf) $f_{X,Y}(u,v) = \frac{1}{2}$ in the area plotted below, and zero else. Obtain:



(a) The constant a for $f_{X,Y}(u,v)$ to be a valid joint pdf.

(b) The marginal probability density function (pdf) of X, $f_X(u)$ for all u.

(c) $E[X]$.

(d) The conditional pdf of Y given X, $f_{Y|X}(v|u)$ for all u, v.