Solution for midterm III

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Problem. 1 Solution:

1) $A = 3/4$.
2) $f_Y(y) = \begin{cases} \frac{1}{4}, 0 < y \leq 1 \\ \frac{3}{4}, -1 \leq y \leq 0 \\ 0, else \end{cases}$.
3) If $0 < y \leq 1$, $f_{X|Y}(x|y) = \begin{cases} 1, 0 < x \leq 1 \\ 0, else \end{cases}$;

   if $-1 \leq y \leq 0$, $f_{X|Y}(x|y) = \begin{cases} 1, -1 \leq x \leq 0 \\ 0, else \end{cases}$;

   otherwise, $f_{X|Y}(x|y)$ does not exist.
4) Half of the cycle with area π is included, so the answer is $\frac{\pi}{4} * \frac{1}{4} + \frac{\pi}{4} * \frac{3}{4} = \frac{\pi}{4}$

Problem. 2 Solution:

1) $\hat{\lambda}_{ML} = k/T$, because calls received in a T minute interval is a $Poi(\lambda T)$ random variable.
2) $P(Poi(2) = 1) = 2e^{-2}$
3) $\binom{5}{3}(1/3)^3(2/3)^2$

Problem. 3 Solution:

1) $[e^0, e^4]$.
2)

$$\begin{aligned} f_Y(v)dv &= P(Y = v) = P(e^{X^2} = v) = P(X^2 = \ln v) = \begin{cases} P(X = \sqrt{\ln v}) + P(X = -\sqrt{\ln v}), e^0 \leq v \leq e^1 \\ P(X = -\sqrt{\ln v}), e^1 < v \leq e^2. \end{cases} \\ &= \begin{cases} 2 * \frac{1}{3} d(\sqrt{\ln v}), e^0 \leq v \leq e^1 \\ \frac{1}{3} d(\sqrt{\ln v}), e^1 < v \leq e^2. \end{cases} = \begin{cases} \frac{1}{3v\sqrt{\ln v}} dv, e^0 \leq v \leq e^1 \\ \frac{1}{6v\sqrt{\ln v}} dv, e^1 < v \leq e^2. \end{cases} \end{aligned}$$

So $f_Y(v) = \begin{cases} \frac{1}{3v\sqrt{\ln v}}, & e^0 \le v \le e^1 \\ \frac{1}{6v\sqrt{\ln v}}, & e^1 < v \le e^2. \\ 0, & else \end{cases}$

Problem. 4 Solution:

1) Choose H_0 if $\sqrt{2} < |X| < \sqrt{\frac{\pi}{2}}e$; otherwise choose H_1.
2) $p_{miss} = 2\left[Q(\sqrt{2}) - Q(\sqrt{\frac{\pi}{2}}e)\right]$
3) Want $\frac{f_0}{f_1} < \frac{\pi_1}{\pi_0}$ to be always true. The maximum value of $\frac{f_0}{f_1}$ is achieved at $\sqrt{\frac{\pi}{2}}e$, which is $\frac{f_0(\sqrt{\frac{\pi}{2}}e)}{f_1(\sqrt{\frac{\pi}{2}}e)} = e^{\frac{\pi}{4}e^2-1}$. So the answer is $e^{\frac{\pi}{4}e^2-1}$.

Problem. 5 Solution: The mean is 50, the std is 5, so the approximate guassian is $N(50, 25)$, and the answer is $P(X \ge 71) = P(X \ge 70.5) \approx P(N(0,1) \ge \frac{70.5-50}{5}) = Q(4.1)$.