Lecture 3

Today: Study more examples on probability space with equally likely outcomes.

Ex 1.4.2) Rolling two fair dice:

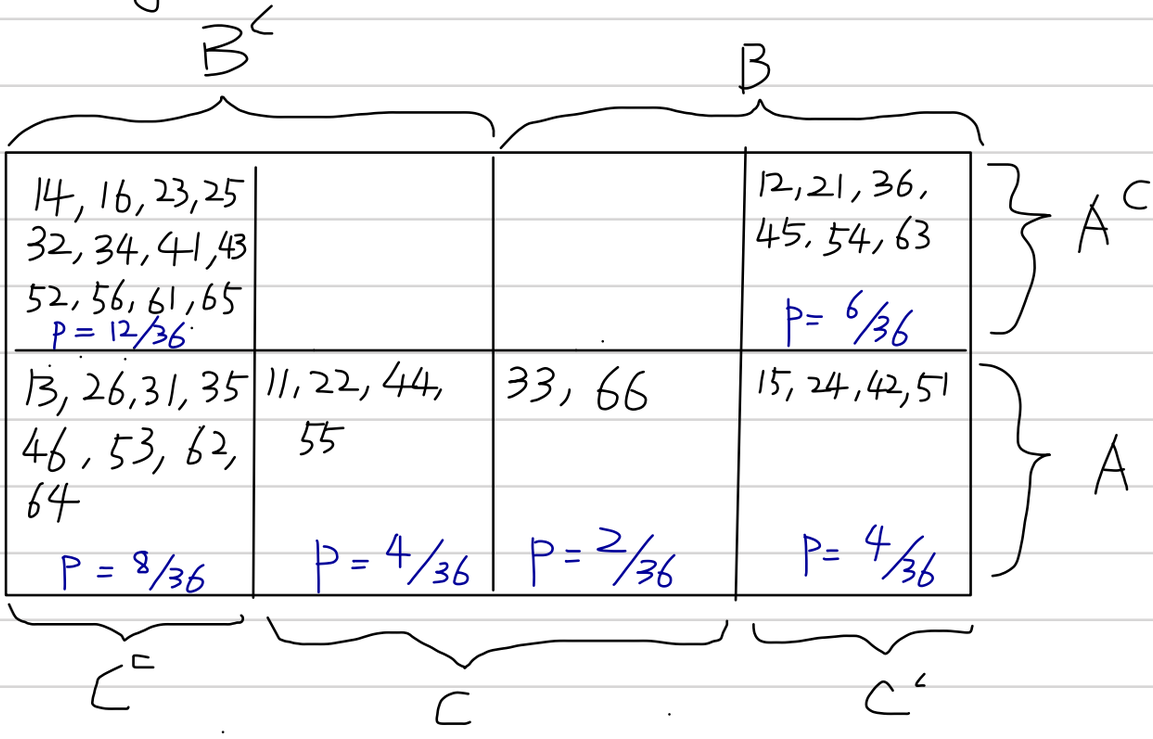
A = "sum is even" B = "Sum is a multiple of three"
B = "the numbers are the same"

Display all outcomes with "Karnaugh Map" and compute P(ABC)?

$\Omega = \{ XY \mid 1 \le X \le 6, \ 1 \le Y \le 6 \}$

* Karnaugh Map: For given events, display the distribution of all outcomes.

e.g.) For given events A BC



The prob of each outcome $= \frac{1}{|\Omega|} = \frac{1}{36}$

$P(ABC) = 2/36 = 1/18.$

Ex 1.4.1) Nine socks in the drawer: six orange & 3 blue
You choose two socks randomly.

Pr(two socks have the same color)?

Sol: Number the socks from $1, 2, \cdots, 9$:
Socks $1, 2, \cdots, 6$ are orange
Socks $7, 8, 9$ are blue

$$\Omega = \{ (i,j) \mid i,j \in \{1,2,\cdots,9\},\ i<j \}$$
$$A = \{ (i,j) \mid i,j \le 6 \text{ or } 7 \le i,j \}$$

$$\Rightarrow |\Omega| = \binom{9}{2} = 36$$

$|A|$ = ways to choose two socks among the orange socks
+ ways to choose two socks among the blue socks

$$= \binom{6}{2} + \binom{3}{2} = 15+3$$

$$P(A) = \frac{15+3}{36} = 1/2.$$

* Binomial Coefficient $\binom{n}{k} = \frac{n!}{(n-k)!k!}$

$(a+b) = a^2 + ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$ why?

Consider the coefficient for $a^k b^{n-k}$

$$(a+b)^n = \overbrace{(a+b)(a+b)(a+b)\cdots(a+b)}^{N \text{ terms}}$$

$a^k b^{n-k} \Rightarrow$ choose k # of a's and $(n-k)$ # of b's

$\Rightarrow \binom{n}{k}$