Assigned: Thursday, June 19
Due: Thursday, June 26
Reading: Ross Chapters 2.1-2.5, 4.1-4.6
Noncredit Exercises: (Do not turn these in) Ross Chapter 1: Problems 1-5,7,9; Theoretical Exercises 4,8,13; Self-Test Problems 1-15. Chapter 2: Problems 3,4,9,10,11-14; Theoretical Exercises 10,20; Self-Test Problems 1-17.

1. Ross, Problem 6, page 59 (6th edition),or page 62 (5th edition), or page 55 (4th edition).

2. Consider an experiment with a finite sample space containing n equally likely outcomes. Thus, there are 2^n different events defined on this sample space.

(a) Show that 2^{n-1} events are comprised of an odd number of outcomes while 2^{n-1} events are comprised of an even number of outcomes. (Zero is an even number)

(b) Find the "average probability" of an event by adding up the probabilities of all 2^n events and dividing the resulting sum by 2^n.

(c) How many of the 2^n different events have probability equal to the average probability that you found in part (b)?

(d) It was noted in class that when a trial of the experiment is performed, exactly 2^{n-1} events occur while the other 2^{n-1} events do not occur. What is the average probability of the 2^{n-1} events that do occur on a given trial of the experiment?

If you cannot solve this problem for general n, solve it (for 50% credit) for the case n = 3 and Ω = {a,b,c}. If you choose to solve only this special case, then, in part (d), assume that outcome b occurs.

3. The experiment consists of picking a student from the set of all UIUC students registered this semester. It is **not** necessary to assume that all students are equally likely to be picked, but you may make this assumption if it makes you feel happier and more confident.

(a) Let A and B denote the events that the student picked has had respectively four years of science (FYS) and calculus in high school. Let $P(A) = 0.45$ and $P(B) = 0.35$. If the probability that the student had neither FYS nor calculus is 0.3, what is the probability that the student had both FYS **and** calculus? What is the probability that the student had FYS but **not** calculus ?

(b) Let C denote the event that the student is registered in ECE 313, and let A and B be as in part (a). Suppose that $P(A \cap B \cap C)=0.002$. What is the probability that the student picked is not registered in ECE 313, but did have both FYS **and** calculus ? If the probability that the student picked is registered in ECE 313, and has had either FYS or calculus (but not both) is 0.004, and if students who had neither FYS nor calculus did not register in ECE 313, what is P(C)?

(c) Using the data given in parts (a) and (b), which of the following probabilities can you compute? It is not necessary to actually compute each probability.

$P(A \cup C)$, $P(A \cup B \cup C)$, $P(A \cup B \cup C^c)$, $P(A^c B^c C^c)$, $P(A^c B C^c)$, $P(ABC^c)$

4. An experiment consists of observing the contents of an eight-bit shift register. Assume that all $2^8 = 256$ bytes are equally likely to be the contents of the shift register.

(a) Let A denote the event that the least significant bit in the shift register is a 1. What is P(A)?

(b) Let B denote the event that the shift register contains 4 0's and 4 1's. What is P(B)?

(c) What is $P(A \cup B)$? What is $P(A \cap B)$? What is the probability that exactly one of the events A and B occurs, i.e. what is $P(A \oplus B)$?

5. Your mother has bought three servings of broccoli and two servings of cauliflower for next week. She chooses the vegetable to be served each day (Monday, Tuesday, Wednesday, Thursday, and Friday) at random (i.e. with equal probability) from those that she still has on hand that day. (On Friday, she has no choice; whatever didn't get served previously is what you are going to get!)

(a) Define an appropriate sample space Ω and state how many outcomes are in Ω.

(b) What is the probability of having broccoli on Monday ?

(c) What is the probability of having broccoli on Monday and Friday ?

(d) What is the probability of having respectively broccoli, broccoli, cauliflower, broccoli, and cauliflower on Monday, Tuesday, Wednesday, Thursday, and Friday ?

6. The manufacturer of a cereal tests samples from the production line to see if the samples snap, crackle, and pop as advertised. Let A, B, and C denote respectively the events that the sample **does not** snap, **does not** crackle, and **does not** pop. The manufacturer's tests show that $P(A) = 0.2$, $P(B) = P(C) = 0.3$, $P(AB \cup BC \cup AC) = 0.3$, $P(ABC) = 0.05$, $P(AB) = 0.1$, and $P(BC) = 2P(AC)$.

(a) Sketch the sample space and indicate on it the events A, B, and C.

(b) What is the probability that the cereal snaps, crackles, and pops ?

(c) Cereal that fails exactly one test is sold to discount supermarket chains at lower prices to be marketed under the brand names Soggies, Blecchies, and Mushies. What is the probability of the sample failing the snap test **only**? the crackle test **only**? the pop test **only**?

7. The experiment consists of picking a letter at random from the word CHATTANOOGA.

(a) Define a sample space with 11 equally likely outcomes. What is the probability that the letter picked is a vowel?

(b) Another way of setting up a probability space is to take $\Omega = \{A,C,G,H,N,O,T\}$ with the outcomes having unequal probabilities. What should be the probabilities assigned to these outcomes so that the probabilities of the various letters are the same as in part (a) ? Now consider picking **three letters** at random (choosing a subset of size 3) from the letters in the word CHATTANOOGA.

(c) What is the probability that the letters chosen can be arranged to form one of the common words CAT, HAT, OAT, TAN and ANT? (Doesn't matter which word is formed)

(d) Repeat part (c) assuming that **sampling with replacement** is being used

(e) Both for sampling with replacement and for sampling without replacement, find the probability that the letters, **as they are picked**, form one of the 5 words of part (c) (doesn't matter which one) **without having to be re-arranged.**

8.(a) Fred, Wilma, Barney, and Betty take turns (in that order) tossing a coin that has $P(\text{Heads}) = p$, $0 < p < 1$. The first one to toss a Head wins the game. Calculate the win probabilities P(F), P(W), P(Ba) and P(Be) of the players and show that

- **(i)** $P(F) > P(W) > P(Ba) > P(Be)$.
- **(ii)** $P(F) + P(W) + P(Ba) + P(Be) = 1$.

(b) $\Omega = \{0, 1, 2, \ldots\}$ is a countably infinite sample space with $P\{n\} = \dfrac{(\ln 2)^n}{2(n!)}$ for all $n \geq 0$.

- **(i)** Verify that $P(\Omega) = 1$ for this probability assignment.
- **(ii)** Show that P(outcome is an even number) = 5/8. Remember: 0 is an even number!

9. The prospectus of GoGoDotCom Inc., an investment management service, states that their goal is to double the value of their clients' investments in a week via day trading of Internet stocks. (The Securities and Exchange Commission insists, as usual, that a disclaimer be included that there is no guarantee that the goal will be met.) The TV commercials proclaim "On average, our clients triple their money in five weeks!" You decide to invest \$32 (hey, you are a student on a tight budget) with GoGoDotCom Inc. for a period of five weeks. Let **X** denote the value (in dollars) of your investment at the end of this period.

Now suppose that GoGoDotCom Inc.has a 50% chance of doubling your investment and 50% chance of losing half your investment. That is, if you invest \$C with them, then, a week later, your investment is equally likely to be worth \$2C or \$C/2. Assume that each week's performance is an independent trial that doubles or halves the value that your investment had at the beginning of the week.

(a) What are the possible values of **X**?

(b) What is the pmf of the random variable **X**?

(c) What is the expected value of **X**? Is the TV commercial an accurate statement?

(d) What is the probability that you will lose money on this investment? i.e. find $P\{\mathbf{X} < 32\}$.

(e) An investment of \$32,000 with GoGoDotCom Inc. would be worth \$1000**X** in five weeks. Assuming that you have the money, would you be willing to make such an investment? Why or why not? Would you be willing to borrow the money from your parents to make the investment? How about borrowing the money from a loan shark?

10. In the game of Chuck-A-Luck often played in MidWestern carnivals and fairs, a player bets money (the stake) on one of the numbers 1, 2, 3, 4, 5, 6. Three fair dice are rolled (so that the sample space has $6^3 = 216$ equally likely outcomes in it). If the number chosen shows up on one, two, or all three of the dice, the player ***wins*** respectively one, two, or three times the money that was staked; and, of course, the original money staked is also returned to the player (the return of the stake money is not counted as part of the winnings). If the chosen number shows up on none of the dice, the player ***loses*** the money that was staked. **X** is the random variable that denotes the amount of money ***won*** for a \$6 bet.

(a) What are the possible values that **X** can take on? Remember that negative values of **X** denote losses.

(b) What is the probability mass function (pmf) of **X**?

(c) What is the expected value of **X**?

(d) ["Always go out a winner"] A player splits his \$6 bet into a \$1 bet on each of the six numbers with the idea that at least one, and possibly as many as three, of his bets will be winners. Let **Y** denote his winnings. What are the values taken on by **Y**? What is the expected value of **Y**? Compare your answer to the expected value of **X** found in part (c).