University of Illinois Spring 2019

ECE 313: Hour Exam I

Wednesday, February 27, 2019
8:45 p.m. — 10:00 p.m.

Name: (in BLOCK CAPITALS) ______________________________

NetID: ______________________________

Signature: ______________________________

Section:
☐ C, 10:00 a.m. ☐ D, 11:00 a.m. ☐ F, 1:00 p.m. ☐ B, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of **five** problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading

1. 20 points ______
2. 24 points ______
3. 22 points ______
4. 16 points ______
5. 18 points ______

Total (100 points) ______

1. **[10+10 points]** An experiment consists of observing the content of an 8-bit register. We assume that all 256 byte values are equally likely to be observed.

   (a) Let B denote the event that the register contains 5 ONEs and 3 ZEROes. What is $P(B)$?

   (b) Let A denote the event that the least significant bit (LSB) is a ZERO. What is $P(A|B)$?

2. **[10+6+8 points]** A 3-faced die is rolled twice. $\omega_1 \in \{1, 2, 3\}$ and $\omega_2 \in \{1, 2, 3\}$ are the outcomes of each roll. The random variable X takes values in the set $\{2\omega_1 + \omega_2\}$.

   (a) Determine the set of values u_i that X can take and the probability mass function $P_X(u_i)$.

   (b) Calculate $E[X]$.

(c) Calculate $\mathrm{Var}(X)$.

3. [**8+10+4 points**] Suppose 50% of total circulating quarter coins in the US are State Quarters (see the figure below for three examples of State Quarters). Suppose that the State Quarters of each State take 1% of the total circulating quarters. Therefore, randomly picking a circulating quarter, the probability that the quarter is an Illinois State (or any other state) Quarter is 1%, and the probability that the quarter is a State Quarter is 50%. (These percentages are made up for exam purposes only.) Bob is collecting the State Quarters by checking every quarter coin he receives in his daily life.

Figure 1: Examples of state quarter coins

(a) What is the expected number of quarters that Bob needs to check to collect 50 Illinois State quarters?

(b) What is the expected number of quarters Bob needs to check to collect two quarters of different states? Present your answer in an irreducible fraction.

(c) What is the expected number of quarters that Bob needs to check to collect a set of state quarters of all 50 states? Present your answer with an integer. (Hint: $1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n} \approx \ln(n)$ and $\ln(50) \approx 3.91$)

4. **[8+8 points]** Suppose a random variable X is uniformly distributed on $\{2, 4, 6, 8, 10, ..., 2n\}$, and another random variable Y has a Poisson distribution with parameter λ.

   (a) We observe $X = 10$. Find the ML estimate of n.

   (b) We observe $Y = 5$. Find the ML estimate of λ.

5. **[12+6 points]** Suppose Z_1, Z_2, Z_3 are i.i.d. Bernoulli random variables with parameter $p = \frac{1}{2}$. The random variable $S = Z_2 Z_3$ if $Z_1 = 1$, and $S = Z_2 + Z_3$ if $Z_1 = 0$.

(a) Find $P(S = 1)$.

(b) Find $P(Z_1 = 1 | S = 1)$.