ECE 313 (Section G)
Homework 8

Due Date: Wednesday, April 12, 11:00 AM in the class

Write your name and NetID on top of all the pages. **Show your work to get partial credit.**

Problem 1 – The number of errors produced by an operating system in a day is described with the random variable *X* that has the following pmf:

No. of Failures	0	1	2	3	4	5	6
Probability	0.18	0.28	0.25	0.18	0.06	0.04	0.01

a) Find the expected number of failures in a day.
b) Find the variance of the number of failures in a day.

Problem 2 – Assume the number of students arriving to the class in an interval of *t* minutes, N_t, is Poisson distributed with mean rate of 2 students per minute. Let *T* represent the inter-arrival times of students.

a) Write the probability mass function (pmf) of *N*.
b) What distribution best describes the inter-arrival times of the students? Write the probability density function (pdf) of *T*.
c) Calculate the expected number of students arriving in a period of 10 minutes.
d) Calculate the expected number of minutes until the arrival of the next student.

Problem 3 – Suppose that a continuous random variable *X* having the following CDF. Find $E[X]$, $E[X^2]$, and $Var[X]$.

$$F(x) = \begin{cases} 0 & x < 0 \\ x^2, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}$$

Problem 4 – There is a dinner party where *n* men check their hats. The hats are mixed up during dinner, so that afterward each man receives a random hat. In particular, each man gets his own hat with probability $\frac{1}{n}$. What is the expected number of men who get their own hat?

i) Let G_i denote the indicator random variable which takes value 1 when the i^{th} man gets his own hat and 0 otherwise. Also let G denote the total number of men who got their own hats. Express G in terms of $G_i's$.
ii) Now using the linearity of expectation, find the expected number of men who get their own hat ($E(G)$).

Problem 5 – Let the random variable X be uniform on the open unit interval $(0, 1)$. Let $Y = g(X) = -ln(X)$.

a) Find the pdf of Y.
b) Calculate the E[Y] using $f_Y(y)$.
c) Now, calculate the E[Y] using $E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x) dx$

Problem 6 – For each of the following systems,

i) write the reliability function $R(t)$,
ii) use the relation $R'(t) = -f(t)$ to derive the failure density function $f(t)$,
iii) use the relation $h(t) = \frac{f(t)}{R(t)}$ to derive the instantaneous failure rate or hazard function $h(t)$,
iv) use the relation $E[T] = \int_0^{\infty} R(t)dt$ to calculate the mean time to failure (MTTF).

a) A component with a constant hazard/ instantaneous failure rate λ.
b) A TMR system composed of three identical components, each with a constant failure rate of λ. The system functions properly when at least 2 out of 3 of the components are functioning.
c) A standby redundant system composed of three components connected in parallel, each with a constant failure rate of λ. Only one component is required to be operative for the system to function properly. Initially the power is applied to only one component and the other two components are kept in a powered-off state (de-energized). When the energized component fails, it is de-energized and removed from operation, and the second component is energized and connected in the second's place, while the third component is still kept in powered-off state. When the second component fails, it is replaced by the third component. (**Hint:** What distribution best describes the time to failure of the system?)

Compare the reliability of the TMR and the standby redundant system in terms of instantaneous failure rate and mean time to failure (MTTF).

Problem 7 – The failure rate of a computer system for onboard control of a space vehicle is estimated to be one of the following functions of time. In each case determine the reliability, $R(t)$, of the component. ($\lambda_0 > 0, \alpha, \beta, \mu, \gamma$ are given constants.)

a) $h(t) = \lambda_0 t$, where $\lambda_0 > 0$ is a given constant.
b) $h(t) = \lambda_0 t^{1/2}$, where $\lambda_0 > 0$ is a given constant.
c) $h(t) = \alpha\mu t^{\alpha-1} + \beta\gamma t^{\beta-1}$