

ECE 313 (Section G)

Homework 6

Due Date: Monday, Mar 13, 11:00 AM in the class

Write your name and NetID on top of all the pages. Show your work to get partial credit.

Problem 1 – If the distribution function of F is given by

$$F(b) = \begin{cases} 0, & b < 0 \\ \frac{1}{2}, & 0 \leq b < 1 \\ \frac{3}{5}, & 1 \leq b < 2 \\ \frac{4}{5}, & 2 \leq b < 3 \\ \frac{9}{10}, & 3 \leq b < 3.5 \\ 1, & b \geq 3.5 \end{cases}$$

calculate the probability mass function of the random variable B .

Problem 2 – The probability density function of X , the lifetime of a certain type of electronic device (measured in hours), is given by

$$f(x) = \begin{cases} \frac{10}{x^2} & x > 10 \\ 0 & x \leq 10 \end{cases}$$

- Find $P\{X > 30\}$.
- What is the cumulative distribution function of X ?
- What is the probability that, of 7 such types of devices, at least 2 will function for at least 20 hours? What assumptions are you making?

Hint: You've previously calculated the probability of device survival, and looked at N-out-of-M systems. Here you must bring those concepts together

Problem 3 – Let X be a random variable with probability density function:

$$f(x) = \begin{cases} kx^2(1 - x^3), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

- Find the value of the constant k .
- What is the cumulative distribution function of X ?
- Find $P\{0.25 < X < 0.5\}$ by using the probability density function.
- Find the probability in part (c) by using the cumulative distribution function $F_X(x)$.

Problem 4 – Lifetimes of VLSI chips manufactured by a semiconductor manufacturer are approximately normally distributed with $\mu = 5 \times 10^6$ h and $\sigma = 5 \times 10^5$ h. A computer

manufacturer requires that at least 95% of a batch should have a lifetime greater than 4×10^6 h. Will the deal be made?

Table 2.3 Area $\Phi(x)$ under the Standard Normal Curve to the Left of x

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5597	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8557	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Problem 5 -

The time (in hours) required to repair a module of a rocket at ACME spacecraft company is an exponentially distributed random variable with parameter $\lambda = 0.5 \text{ hr}^{-1}$

Before the launch, the technical team found out that the solid-fuel booster module has been damaged and needs to be repaired.

1. What is the probability that the repair time exceeds 2 hours
2. What is the conditional probability that a repair time takes at least 10 hours, given that the technical team has already been repairing for 8 hours.

Problem 6 -

A system consists of blocks A, B, C, D and E connected as shown in the figure. The reliability of each block is exponentially distributed with parameter λ . Find the reliability of the system.

