ECE 313 (Section G)
Homework 4

Due Date: Wednesday, March 1, 11:00 AM in the class

Write your name and NetID on top of all the pages. Show your work to get partial credit.

Problem 1 – For each of the following cases, determine whether the random variable is discrete or continuous. For the discrete random variables: (i) Describe the probability mass function (pmf) by finding the set of values that the random variable might take and their respective probabilities (The solution to Parts (a) and (b) are given as examples for you).

a) T is the lifetime of a light bulb.
Solution: T is a continuous random variable.

b) A certain manufacturing firm produces chipsets that are non-defective with the probability of $p = 0.9$. A quality control crew randomly picks chipsets and tests them for defects. He is asked to repeat this process until the first defective product is found.

X is the number of defective chipsets in the sample of 10 randomly chosen chips

N is the number of trials until the first defective chipset is found

Solution: X is a discrete random variable.
It takes values of $\{0, 1, 2, 3, ..., 10\}$ with the following probabilities:
$P\{X = 0\} = p(0) = \binom{10}{0}(0.9)^{10}(0.1)^{0} \approx 0.3487$
$P\{X = 1\} = p(1) = \binom{10}{1}(0.9)^{9}(0.1)^{1} \approx 0.3874$
$\vdots$
$P\{X = 10\} = p(10) = \binom{10}{10}(0.9)^{0}(0.1)^{10} = (0.1)^{10}$
In general $P\{X = k\} = p(k) = \binom{10}{k}(0.9)^{10-k}(0.1)^{k} \quad k = 0,1,...,10$

N is also a discrete random variable.
It takes values of $\{1, 2, 3, ...\}$ with the following probabilities:
$P\{N = 1\} = p(1) = p = 0.1$
$P\{N = 2\} = p(2) = (1 - p)p = (1 - 0.1)(0.1) = 0.09$
$P\{N = 3\} = p(3) = (1 - p)^{2}p = (1 - 0.1)^{2}(0.1) = 0.081$
$\vdots$
In general $P\{N = n\} = p(n) = (1 - p)^{n-1}p \quad n = 1, 2, ...$

c) Let Y represent the difference between the number of heads and the number of tails obtained when a fair coin is tossed 2 times.

d) L is the lifetime of a valve in a randomly selected cooling cabinet.

e) An amateur mind game player has accepted a challenge from a computer program. The challenge consists of three rounds (with different games for each round) and the player is awarded $1,000 for each round that the player has won. In addition, the final winner (who wins more than two out of the three rounds) is awarded additional $3,000. A preliminary analysis has shown that the possibility of the player winning the computer program is 0.55 for the first round, 0.7 for the second round and 0.4 for the third round. Let X be a random variable on the total award that the player can win. Assume that all three rounds are played regardless of the results.

f) Assume we have weather forecasting application with a prediction accuracy of 85%. Let W be a random variable representing the number of days in a week that the application successfully predicts the weather.

g) Let A represent an analog signal received by an analog to digital converter.

h) T represents the time between two students arriving at Siebel 1109 for ECE313.

Problem 2 – Consider the random variable X with pmf $P(X = i) = 2^{-i}$ for $i \geq 1$.

a) Sketch the pmf (given above) for X.

b) Show that the given pmf is a valid pmf. Hint: what condition needs to be satisfied for a function to be a valid pmf?

c) Calculate $P(X \leq 4)$.

d) Calculate $P(X > 4)$.

e) Calculate $P(X < 1)$.

f) Calculate $P(|X-5| \leq 0.1)$.

g) Evaluate the following expression:

$$\sum_{k=6}^{\infty} P(X = k)$$

Problem 3 – For each of the following, please describe whether it is a valid CDF or not. Justify your answers.

a $$F(x) = \begin{cases} 0, & if\ x < 0 \\ 4x^4 - 3x^2 & if\ 0 \leq x < 1 \\ 1 & if\ x \geq 1 \end{cases}$$

b $F(x) = \begin{cases} 0, & if\ x < 0 \\ 0.5 + e^{-x} & if\ 0 \le x < 3 \\ 1 & if\ x \ge 3 \end{cases}$

c $F(x) = \begin{cases} 0, & if\ x < -1 \\ x^2 & if -1 \le x < 1 \\ 1 & if\ x \ge 1 \end{cases}$

Problem 4 – Identical computer components are shipped in boxes of 5 (A box contains 5 components). About 15% of components have defects. Boxes are tested in a random order.

a) What is the probability that a randomly selected box has only non-defective components?

b) What is the probability that at least 8 of randomly selected 10 boxes have only non-defective components?

c) What is the distribution of the number of boxes tested until a box without defective components is found?

Problem 5 – Assume that the number of jobs arriving to the Blue Waters super-computer in an interval of *t* seconds is Poisson distributed with parameter $\lambda = 0.5$. Compute the probabilities of the following events:

a) Exactly 3 jobs will arrive during a 10s interval.

b) More than 10 jobs arrive in a period of 20s.

c) The number of job arrivals in an interval of 10s duration is between two and four.

d) Given that 10 jobs arrive in a period of 30s, what is the conditional probability that 3 jobs arrived in the first 10s?

Hint: Use the Bayes theorem to calculate the conditional probability. Note that the probability of 3 jobs arriving in the first 10s, given that 10 jobs arrived in 30s, equals to the probability of 3 jobs arriving in the first 10s and 7 jobs arriving in the remaining 20s. Also note that the number of arrivals in different time intervals are independent from each other.