University of Illinois **Fall 2016**

ECE 313 (Section X)
Homework 2
No Due Date.

Problem 1 - A binary communication channel carries data as one of the two types of signals denoted by 0 and 1. As a result of noise, a transmitted 0 is sometimes received as a 1 and a transmitted 1 is sometimes received as a 0. For a given channel, assume a probability of 0.9 that a transmitted 0 is correctly received as a 0 and a probability of 0.85 that a transmitted 1 is received as a 1. Further assume a probability of 0.45 of transmitting a 0. If a signal is sent, determine the:

a) Probability that a 0 is received.
b) Probability that a 1 was transmitted, given that a 1 was received.
c) Probability that a 1 was transmitted, given that a 0 was received.
d) Probability of an error.

Problem 2 – Prove or give counterexamples to the following statements:

a) If E is independent of F, and E is independent of G, then E is independent of $F \cup G$.

b) If E is independent of F, and E is independent of G, and $F \cap G = \emptyset$, then E is independent of $F \cup G$.

Problem 3 – A total of 46 percent of the voters in a certain city classify themselves as independents, whereas 30 percent classify themselves as liberals, and 24 percent classify themselves as conservatives. In a recent local election, 35 percent of the independents, 62 percent of the liberals, and 58 percent of the conservatives voted. A voter is chosen at random. Given that this person voted in the local election, what is the probability that he or she is:

a) An independent

b) A liberal

c) A conservative

d) What fraction of voters participated in the local election?

Problem 4 – An AND gate takes two inputs A and B that are independent. $P(A = 0) = P(A = 1) = P(B = 0) = P(B = 1) = 0.5$. The inputs are gated to a clock so that they only change on the falling edge of the clock signal. On a given falling edge of the clock, what is the probability that the output C of the AND gate switches (i.e., the output was previously a 0 and transitioned to a 1 when the new inputs A and B are applied, and vice versa)? Note that both inputs A and B can change on the falling clock edge.

Problem 5 – Box A has 3 white balls and 2 black balls. Box B has 2 white balls and 2 black balls. A ball is drawn at random from Box A and transferred to Box B without looking at the ball's color. Then a ball is drawn at random from Box B. What is the probability that the ball drawn from Box B is black?

Problem 6 – How can 20 balls, 10 white and 10 black, be put into two urns so as to maximize the probability
of drawing a white ball if an urn is selected at random and the ball is drawn at random from it?

Problem 7 – Recall the discussion from lecture concerning physical vs. stochastic independence. Consider an XOR gate with two inputs A and B. Let event A represent a logical 1 on input A, event B a logical 1 on input B, and event C a logical 1 on the output C of the XOR gate. Assume that A and B are independent.

a) Write a set expression for C in terms of A and B. What is P(C)?

b) Take P(A) = P(B) = 0.5. Are A and C stochastically independent?

c) Take P(A) = P(B) = 0.7. Are A and C stochastically independent?

d) Take P(A) = P(B) = 0.5 + e, where e is some small positive number. Are A and C stochastically independent?