Problem 1 –

a. We enter the for loop 13 times, and in each loop entry we have 2 possible cases based on the value of variable \( p \):
   Sample space consists of \( 2^{13} \) combinations of ‘A’ s and ‘B’ s.
   Number of ways exactly 4 ‘A’ s can occur: \( \binom{13}{4} \)

b) There are three positions to fill: the units, the tens, and the hundreds. For the number to be odd, there are 5 options for the units position: \{1, 3, 5, 7, 9\}:
   - If the unit position is filled with \{1, 3, 5, 7, 9\},
     o For the hundreds position, we cannot use 0 and we cannot use the number that was used in units position. So we are left with 8 choices.
     o For the tens position, we cannot use the numbers used in the hundreds or the units positions, so again we are left with 8 choices.
     o Hence, we have \( 8 \times 8 \times 5 \) choices = 320.
   So the sample space size will be 320.

c) The digit 8 can be any of these positions: units, tens, or hundreds. So we have 3 options. The two remaining positions can be filled in \( 9 \times 9 \) ways. Also, there are 4 options (0,1,2,3) to fill the position of thousands. Therefore, there are \( 3 \times 9 \times 9 \times 4 = 972 \) such numbers.

   \[ \begin{align*}
   (4, 9, 9, 1) & - 324 \\
   (4, 9, 1, 9) & - 324 \\
   (4, 1, 9, 9) & - 324
   \end{align*} \]

d) - First student has \( n \) options for choosing the first book to borrow, \( n-1 \) for the second book, \( n-2 \) for the third book and \( n-3 \) for the fourth book. Therefore first student has \( \binom{n}{4} \) options.
   - As the first student has rented 4, second student now has \( \binom{n-4}{4} \) options
   - Now that 8 of the \( n \) books are already borrowed, the third student has \( \binom{n-8}{4} \) options left.
   Therefore, the sample size is \( \binom{n}{4} \times \binom{n-4}{4} \times \binom{n-8}{4} \).

Problem 2 – M – ‘malicious’, B – ‘benign’
The sample space for this experiment consists of ten items, each either being M or B. Thus, there are \( 2^{10} = 1024 \) elements in \( S \). To have exactly 4 consecutive ‘M’ s, the following pattern must be present in each trial: \{B, M, M, M, M, B\}

The question then becomes how many elements in the sample space contain this pattern as a subset. This subset can appear as follows:
\{B, M, M, M, M, B, X, X, X, X\}
\{X, B, M, M, M, B, X, X, X\}
where X can be either a M or B. Each possibility contains four X’s, and the number of elements for each possibility is 2^4 = 16. So we have 5 × 16 outcomes.

In addition, we must also take into account the fact that the four M’s can appear at the beginning and end of the history as well:

There are 2×2^5 = 64 outcomes that match this pattern. But we are counting the following patterns twice: {M, M, M, M, B, M, M, M, M}, \{B, M, M, M, B, M, M, M, M\}.

So, there are 5 × 16 + 64 − 2 = 142 different elements in S in which the desired subset appears, resulting in a probability of 142/1024.

**Problem 3**

a) \( S = \{(A, B, C, D, PS), (A, B, C, D, PS'), (A, B, C, D', PS), (A, B, C, D', PS')... (A', B', C', D', PS')\} \)

\(|S| = 2^5 = 32 \)

b) For the control system to be active, at least one of A or B or C must be working, D must be working and PS must be working

\[ E = \{(A, B, C, D, PS), (A, B, C', D, PS), ..., (A', B', C, D, PS)\} \]

\[ P(A) = (1-p)(1-p)(1 - pp) \]

**Problem 4** — Sample space of the problem consists of the number of ways that you can choose 4 fruits from 9 bananas, 5 apples, 3 peaches, and 4 oranges:

\[ \text{Sample space size} = \binom{9+5+3+4}{4} = \binom{21}{4} \]

a) If you pick one from each group of 9, 5, 3 and 4 fruits:
\[ P = \binom{9}{1} \times \binom{5}{1} \times \binom{3}{1} \times \binom{4}{1} \times \binom{21}{4} \]

b) If you pick 2 apples of 5 apples and then another 2 peaches of the 3 peaches available:

\[ P = \frac{\binom{5}{2} \times \binom{3}{2}}{\binom{21}{4}} \]
c) The possibility can be written as:

\[
P = 1 - \left( \frac{16}{4} + \frac{17}{4} - \frac{12}{4} \right)
\]

\[
= 1 - \left( \frac{21}{4} \right)
\]

\[
= 1 - \frac{21}{4}
\]

d) There are 3 possibilities – zero bananas, one banana, 2 bananas

\[
P = \left( \frac{12}{4} \right) + \left( \frac{9}{1} \frac{12}{3} \right) + \left( \frac{9}{2} \frac{12}{2} \right)
\]

\[
\frac{21}{4}
\]

**Problem 5** - An experiment consists of observing the contents of an 16-bit register. We assume that all 256 byte values are equally likely to be observed.

a) Any arbitrary 15-digit binary number concatenated with a ZERO is a member of event A. Thus, there are \(2^{15}\) such numbers out of \(2^{16}\) 16-digit numbers. Since all numbers are equally probable, we have:

\[
P(A) = \frac{2^{15}}{2^{16}} = 0.5
\]

b) \[
P(B) = \left( \frac{16}{11} \right) \left( \frac{1}{2} \right)^{11} \left( \frac{1}{2} \right)^{5}
\]

The first term indicates the number of different combinations that satisfy the condition that 16 out of 16 digits are ZEROs. The second term indicates the probability that these 11 digits are ZEROs. The third term indicates the probability that the remaining 5 digits are ONEs.

c) The intersection of event A and B can be described as: the last digit must be a ZERO, and out of the first 15 digits, there are 5 ONEs and 10 ZEROs. Thus, the probability of the intersection is:

\[
P(A \cap B) = \left( \frac{1}{2} \right) \left( \frac{15}{10} \right) \left( \frac{1}{2} \right)^{10} \left( \frac{1}{2} \right)^{5}
\]

Calculating the probability of at least A or B occurring is then straightforward:

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.07 - 0.05 = 0.52
\]

d) The probability that exactly one of A and B occur can be calculated by:

\[
P(A \oplus B) = P(A \cup B) - P(A \cap B) = P(A) + P(B) - 2P(A \cap B) = 0.5 + 0.07 - 0.09 = 0.48
\]
e) What is the probability of at least one of A or B does not occur?

\[ P(A^c \cup B^c) = 1 - P(A \cap B) = 1 - (0.05) = 0.95 \]

**Problem 6 –**

a) There are two players in the game, and each player has 2 possible actions: confess (C) or deny (same as remaining silent) (D). So the sample of space of the problem consists of:

\[ S = \{(C, C), (C, D), (D, C), (D, D)\} \]

b) 0.6, because A is confessing and B won’t confess with a probability of 0.6.

c) 0.4, because A is confessing and B will confess with a probability of 0.4.