ECE 313 (Section G)
Homework 10
Due Date: Wed, Apr 26, in the class

Write your name and NetID on top of all the pages. **Show your work to get partial credit.**

Covariance and Correlation

Problem 1 – Suppose n fair dice are independently rolled. Let:

$$X_k = \begin{cases} 1 & if\ 1\ shows\ on\ the\ k^{th}\ die \\ 0 & else \end{cases} \qquad Y_k = \begin{cases} 1 & if\ 2\ shows\ on\ the\ k^{th}\ die \\ 0 & else \end{cases}$$

Let $X = \sum_{k=1}^{n} X_k$, which is the number of one's showing, and $Y = \sum_{k=1}^{n} Y_k$, which is the number of two's showing. Note that if a histogram is made recording the number of occurrences of each of the six numbers, then X and Y are the heights of the first two entries in the histogram.

a) What distribution best describes the random variables X_k and Y_k?
b) Find $E[X_k]$ and $E[Y_k]$.
c) Use $E[X_k]$ and $E[Y_k]$ to find $E[X]$ and $Var(X)$.
d) Find $Cov(X_i, Y_j)$ if $1 \leq i \leq n$ and $1 \leq j \leq n$
(**Hint:** Are X_i, and Y_j independent when $i \neq j$? What if $i = j$?)
e) Use $Cov(X_i, Y_j)$ to find $Cov(X, Y)$.
f) Find the correlation coefficient $\rho_{X,Y}$. Are X and Y positively correlated, uncorrelated, or negatively correlated?

Problem 2 – Consider two random variables X and Y:

a) If $\text{Var}(X + 2Y) = \text{Var}(X - 2Y)$, are X and Y uncorrelated? Are they independent? Why?
b) If $\text{Var}(X) = \text{Var}(Y)$, are X and Y uncorrelated? Why?

Hypothesis Testing

Problem 3 – Given the following likelihood matrix, specify the ML and MAP decision rules and find $p_{false-alarm}$, p_{miss}, and p_e for both of them. Assume that: $\pi_0 = 0.3$.

	X = 0	X = 1	X = 2	X = 3	X = 4
H_1	0.00	0.10	0.24	0.34	0.32
H_0	0.08	0.15	0.31	0.31	0.15

Problem 4 – We are interested in detecting Denial of Service (DOS) attacks to a Web server based on the incoming traffic that is observed at a given second.

- Suppose that usually the number of requests, X, arriving at the Web Server from one client in an interval of 1 second is Poisson distributed with parameter $\lambda = 3$.
- In case of a DOS attack, an attacker sends a total of 100 malicious requests per second, from which X would successfully pass the firewall and arrive to the Web Server. Suppose the probability that a malicious request passes through the firewall and arrives to the server is $p = 0.04$.

A decision rule is needed to decide, based on observation of X, whether an attack occurred (hypothesis H_1) or not (hypothesis H_0).

a) Write the conditional pmf of observation X given hypotheses H_0.

b) Write the conditional pmf of observation X given hypotheses H_1. (**Hint:** What distribution best describes the number of malicious requests that successfully arrive to the Web Server?)

c) Describe the ML decision rule in the form of likelihood ratio tests (LRT).

(**Hint:** For which values of X an attack (H_1) is declared by the ML decision rule?)

d) Describe the MAP decision rule in the form of likelihood ratio tests (LRT), under the assumption that receiving normal requests is a priori three times more likely than receiving attacks (i.e. $\frac{\pi_0}{\pi_1} = 2$).

e) Find $p_{false-alarm}$, p_{miss}, and p_e for each of the ML and MAP decision rules.

f) Compare the performance of the ML or MAP decision rules.