University of Illinois Spring 2015

ECE 313: Hour Exam I

Wednesday, March 4, 2015
7:00 p.m. — 8:15 p.m.
MSEB 100, ECEB 1002, ECEB 2017

Name: (in BLOCK CAPITALS) ____

NetID: ____

Signature: ____

Section:
☐ E, 9:00 a.m. ☐ C, 10:00 a.m. ☐ D, 11:00 a.m. ☐ F, 1:00 p.m. ☐ B, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. No electronic equipment (cell phones, etc.) allowed.

The exam consists of six problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading	
1. 20 points	______
2. 15 points	______
3. 15 points	______
4. 15 points	______
5. 20 points	______
6. 15 points	______
Total (100 points)	______

1. **[20 points]** The two parts of this problem are unrelated.

   (a) Suppose that you have five distinct bowls, each can hold up to two oranges. If you have four indistinguishable oranges, how many ways are there to place the oranges in the bowls?

   (b) Suppose that five men and five women are invited to have dinner at a round table. How many seating arrangements are there if there must not be two men nor two women seating next to each other? (NOTE: a seating plan at a round table is invariant with respect to rotation.)

2. **[15 points]** Two random bits (equally likely to be 0 or 1) are drawn independently from a file. Let X denote the sum of the two bits.

   (a) Find $E[X]$

   (b) Find $\mathrm{Var}(X)$.

   (c) Find $\mathrm{Var}(3X + 2)$.

   (d) Find $E[\frac{1}{1+X}]$.

3. **[15 points]** A fair die is rolled. Let X be the number of rolls required until 2 or 3 show for the first time.

   (a) Find $E[X]$.

   (b) Find $P(X = 5 \mid X > 3)$.

   (c) Find $E[X \mid X > 3]$.

4. **[15 points]** A hundred cars participate in a race. The probability that a car breaks down and drops out of the race is 0.01.

   (a) Find the probability that all cars finish the race.

   (b) Use Poisson approximation to find the probability that exactly one car breaks down. Leave your answer in terms of e.

5. **[20 points]** Peter has a biased coin, which has $P\{heads\} = 1/3$. He flips the coin twice. Let X be the number of heads that show. After Peter flips the coin twice, he rolls a fair die X times. Let Y be the number of times that 3 shows.

(a) Compute the probability that $Y = 1$ (i.e., compute $P\{Y = 1\}$).

(b) Suppose Peter rolled 3 once. Compute the probability that X was 2 (i.e., compute $P\{X = 2 \mid Y = 1\}$).

6. **[15 points]** A standard deck of cards contains 52 cards (`A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K` of each suit), while a piquet deck contains only 32 cards (`A, 7, 8, 9, 10, J, Q, K` of each suit).

   (a) If you are given a card drawn at random from an unknown deck, formulate the ML decision rule whether the deck is the standard one ($\mathbf{H}_o$) or the piquet deck ($\mathbf{H}_1$).

   (b) Suppose that the chances to see a piquet deck in the US are one in three. Formulate the corresponding MAP decision rule.