University of Illinois Spring 2014

ECE 313: Hour Exam II

Monday, April 14, 2014
7:00 p.m. — 8:15 p.m.
Rooms 141 (Sections E,C, and those in D with last names A-K) and 151 Loomis Lab

1. **[18 points]** A nuclear football is a briefcase from which the president of a nation (to remain unnamed) can launch a nuclear attack. We will consider a nuclear football with two buttons, each of which will close the contacts of one switch, i.e., if you hit a button, you close a path between the two ends of the switch. The switches are connected either in a series or in a parallel configuration, and for a nuclear attack to be launched, there needs to be a path between the two ends of the configuration.

   While the switches are extremely reliable, there is nonzero probability, p, that each switch will fail, i.e., without pushing the button, the contacts will close. In the remainder, assume that the switches fail independently.

   (a) (6 points) If the switches are connected in parallel: (i) find the exact probability that the nuclear football will activate an unwanted nuclear attack due to switch failure, and (ii) use the union bound to provide an upper bound on the aforementioned probability.

   (b) (6 points) If the switches are connected in series, compute the exact probability that the nuclear football will activate an unwanted nuclear attack because failures in the switches.

   (c) (6 points) Which design is better, parallel or series, assuming $p << 1$? Briefly explain your reasoning. Take into account the strengths and weaknesses of each approach.

2. **[12 points]** The random variable X has a $\mathcal{N}(\mu, \sigma^2)$ distribution. Determine the following probabilities (Simplify your answers as much as possible. If appropriate, use the Φ or Q function.)

   (a) (6 points) $P\{X^2 + 1 \leq -203.57\}$

   (b) (6 points) $P\{X^2 - 2X \leq 0\}$

3. **[18 points]** Buses arrive at a station starting at time zero according to a Poisson process $(N_t : t \geq 0)$ with rate λ, where N_t denotes the number of buses arriving up to time t.

   (a) (6 points) Given exactly one bus arrives during the interval $[0, 3]$, find the probability it arrives before $t = 1$. Show your work or explain your reasoning.

   (b) (6 points) What is the probability that there is a bus arriving exactly at time $t = 1$?

   (c) (6 points) Suppose Alice arrives at the station at time T which is uniformly distributed on $[0, 1]$ and independent of the bus arrival process. She decides to wait one unit of time. If no bus arrives while she waits, she walks home. What is the probability Alice ends up walking home?

4. **[18 points]** An electric heating element produces W watts of heat, where $W = I^2R$, I is the current through the element in amps, and R is the resistance in ohms. Suppose $R = 2$ and I is modeled as a random variable uniformly distributed over the interval $[1, 3]$.

   (a) (6 points) Find $P\{W \leq 2.88\}$. Simplify your answer as much as possible.

   (b) (6 points) Find $E[W]$.

   (c) (6 points) Find a formula for the CDF of W, F_W. Be sure to specify $F_W(c)$ for all real values of c. (You do *not* need to find the pdf.)

5. **[10 points]** Suppose T is the lifetime of an electric turbine measured in years. It is assumed to be a continuous type random variable with the failure rate function:
$h(t) = \begin{cases} \theta & 0 \le t \le 5 \\ 4\theta & t > 5 \end{cases}$ for some $\theta > 0$.

   (a) Express $P\{T \ge 6\}$ in terms of θ. Simplify your answer as much as possible.

   (b) Suppose it is observed that $T = 4.00$. Find the corresponding maximum likelihood estimate, $\widehat{\theta}_{ML}$.

6. **[24 points]** Let X and Y be jointly continuous random variables with joint pdf:

$$f_{X,Y}(u,v) = \begin{cases} 0.5 & 0 \le u \le 1, 0 \le v \le 1 \\ 0.5 & 1 \le u \le 2, 1 \le v \le 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) (6 points) Are X and Y independent? (Explain your answer.)

   (b) (6 points) Find the marginal pdf $f_X(u)$. Be sure to specify it for $-\infty < u < \infty$.

   (c) (6 points) Find the conditional pdf $f_{X|Y}(u|1.5)$. Be sure to specify it for $-\infty < u < \infty$.

   (d) (6 points) Find the numerical value of $P\{Y \ge 4X^2\}$.