University of Illinois Spring 2013

ECE 313: Hour Exam I

1. (a)

$$P(AB) = \boxed{0.15}$$

$$P(AB^c) = \boxed{0.15}$$

$$P(A^cB) = \boxed{0.55}$$

$$P(A^cB^c) = \boxed{0.15}$$

(b)

$$P(AC) = \boxed{0.12}$$

$$P(A^cC) = \boxed{0.28}$$

$$P(BC|A) = \boxed{0.2}$$

(c)

$$\frac{9}{12} \leq P(B|D) \leq \frac{11}{12}.$$

2. (a) By the law of total probability, the solution is $\frac{1}{6}\frac{3}{5} + \frac{5}{6}\frac{1}{5} = \frac{4}{15}$.

   (b) By Bayes formula, the solution is $\frac{\frac{1}{6}\frac{3}{5}}{\frac{4}{15}} = \frac{3}{8}$.

3. (a) $\mathbb{N}$ is a geometric random variable with parameter $\frac{1}{6}$. So the mean is 6.

   (b) The event $\mathbb{N} \geq 4$ is the same as $\mathbb{N} > 3$. From the memoryless property of the geometric random variable, you may guess that conditioned on $\mathbb{N} > 3$ the random variable is still geometric with parameter $\frac{1}{6}$, so the mean is simply $3 + 6 = 9$. Below is the detailed and formal calculation.

   Conditioned on the event $\mathbb{N} > 3$, the random variable $\mathbb{N}$ has the range $\{4, 5, 6, 7, \ldots\}$.

The conditional pmf, for k in the range, is

$$\begin{aligned}
P(N = k|N > 3) &= \frac{P(N = k)}{P(N > 3} \\
&= \frac{\frac{1}{6}\left(\frac{5}{6}\right)^{k-1}}{\left(\frac{5}{6}\right)^{3}} \\
&= \frac{1}{6}\left(\frac{5}{6}\right)^{k-4}.
\end{aligned}$$

Now the conditional mean is

$$\begin{aligned}
\mathsf{E}[\mathbb{N}|\mathbb{N} > 3] &= \sum_{k=4}^{\infty} kP(\mathbb{N} = k|\mathbb{N} > 3) \\
&= \sum_{k=4}^{\infty} k\frac{1}{6}\left(\frac{5}{6}\right)^{k-4} \\
&= \sum_{\ell=1}^{\infty} (\ell + 3)\frac{1}{6}\left(\frac{5}{6}\right)^{\ell-1} \quad ; \quad \ell = k - 3 \\
&= \left(\sum_{\ell=1}^{\infty} \ell\frac{1}{6}\left(\frac{5}{6}\right)^{\ell-1}\right) + \left(\sum_{\ell=1}^{\infty} 3\frac{1}{6}\left(\frac{5}{6}\right)^{\ell-1}\right) \\
&= 6 + 3 \\
&= 9.
\end{aligned}$$

4. (a) This problem is worked out in detail in your text book. The outage probability is $\frac{1}{2}$.
   (b) $\mathbb{X}$ can take on the values 0, 10 or 20.

5. (a) There are only two possibilities out of 36 in the first two rolls. Since all the possibilities are equally likely, the desired probability is simply $\frac{1}{18}$.
   (b) Since there are exactly 3 rolls the total possibilities of the numbers that show up is 216. Now the last roll must have yielded either a 1 or 2. For each of these choices, there are exactly 9 possibilities of the first two rolls. Since all the possibilities are equally likely, the desired probability is $\frac{18}{216} = \frac{1}{12}$.