ECE 313: Hour Exam I

1. [**25 points**] Two events, A and B, occur with the following probabilities:

$$\begin{aligned} P(A) &= 0.3 \\ P(B) &= 0.7 \\ P(B|A) &= 0.5 \end{aligned}$$

(a) [**8 points**] Find the following:

$$P(AB) = \boxed{}$$

$$P(AB^c) = \boxed{}$$

$$P(A^cB) = \boxed{}$$

$$P(A^cB^c) = \boxed{}$$

(b) [**9 points**] Event C occurs with probability $P(C) = 0.4$, independent of events A and B. Find the following:
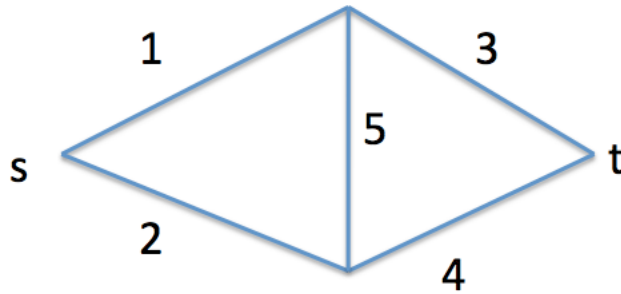
$$P(AC) = \boxed{}$$

$$P(A^cC) = \boxed{}$$

$$P(BC|A) = \boxed{}$$

(c) [**8 points**] Event D is mutually exclusive of event A ($P(AD) = 0$), and occurs with probability $P(D) = 0.6$. You do not have enough information to find $P(B|D)$, but you have enough information to find pretty tight upper and lower bounds on this quantity. Find these bounds.

$$\boxed{} \le P(B|D) \le \boxed{}$$

2. [**20 points**] Anna belongs to a book club which meets once every weekend to discuss one book (which could be either a fiction book or a nonfiction book). The book club picks a book at random, with fiction books being five times as likely as nonfiction books to be chosen. Anna decides to attend the weekend club meeting with probability $\frac{1}{5}$ if the chosen book is fiction and with probability $\frac{3}{5}$ if the chosen book is nonfiction.

   (a) [**10 points**] What is the probability p that Anna attends the weekend club meeting?

   (b) [**10 points**] Given that Anna attended the book club meeting, what is the probability q that the chosen book is nonfiction?

3. [**20 points**] A fair die is rolled until the number 6 first appears. Let $\mathbb{N}$ be the number of rolls, including the last roll.

   (a) [**5 points**] Find the mean of $\mathbb{N}$.

   (b) [**15 points**] Given that we have rolled at least 4 times, what is the expected number of rolls?

4. [**20 points**] Consider the following network between s and t with each of the five links failing independently with probability $\frac{1}{2}$.

   (a) [**10 points**] Find the probability of *outage* for communication between s and t.

   (b) [**10 points**] Suppose each link has capacity 10 units. Let $\mathbb{X}$ denote the (random) communication capacity of the network. What range of values can $\mathbb{X}$ take?

5. [**15 points**] A fair die is rolled until the numbers 1 and 2 show up (not in any particular order or in succession). Let M be the number of rolls.

   (a) [**5 points**] Find $P\{M = 2\}$.

   (b) [**10 points**] Find $P\{M = 3\}$.