

ECE 313: Problem Set 8

Continuous-type random variables, uniform distribution, exponential distribution

Due: Friday March 18 at 4 p.m.

Reading: 313 Course Notes Sections 3.2–3.4

1. [Continuous-type random variables]

In each case given below, check if $f(x)$ is a valid pdf. For those which are not valid pdfs, state at least one property of pdfs which is not satisfied. Also indicate if there exists a constant c such that $cf(x)$ is a valid probability density, and if so, find the value of c . The function $f(x)$ is equal to zero outside of the ranges indicated.

- (a) $f(x) = |x|$ for $|x| < 1$.
- (b) $f(x) = \frac{2}{3}(x - 1)$ for $0 < x < 3$.
- (c) $f(x) = x^{-1}$ for $1 \leq x < \infty$.
- (d) $f(x) = \ln(x)$ for $0 < x < 1$.
- (e) $f(x) = e^{-2x}$ for $0 < x < \infty$.
- (f) $f(x) = x^2e^{-x}$ for $0 \leq x < \infty$.
- (g) $f(x) = xe^{-x^2}$ for $0 \leq x < \infty$.

2. [Continuous-type random variables]

The pdf of a random variable X with mean $3/2$ and variance $1/12$ is given by:

$$f_X(u) = \begin{cases} au^2 + bu + c, & 1 \leq u \leq 2 \\ 0, & \text{else.} \end{cases}$$

- (a) Find a , b and c .
- (b) Find $F_X(u)$ for all u .
- (c) Find the third moment of X .

3. [Continuous-type random variables]

Let Y be a random variable that denotes the daily requirements (in pounds) for fertilizer at a farm, independently of the requirements any other day. The pdf of Y is given by $f_Y(y) = a(4 - (2 - y)^2)$ for $0 \leq y \leq 4$ and some constant a . Let C denote the capacity (in pounds) of the farm's fertilizer storage tank, which is refilled daily. The farmer is interested in the random variable X , equal to the amount (in pounds) of fertilizer used in a given day. Note that the amount of fertilizer used cannot exceed the tanks capacity, i.e., $X = \min(Y, C)$. Assume that the cost of each pound of fertilizer is \$0.32.

- (a) Find the constant a that makes the density f_Y a valid pdf.
- (b) Suppose, only for this part of the problem, that $C = 2$ and that it is observed on a particular day that $Y = 1.105$. Can the farmer satisfy the fertilizer requirements that day?
- (c) Find, as a function of C , the probability that the fertilizer requirements are satisfied on any given day.
- (d) What is the minimum value of C to ensure that the probability that the fertilizer requirements exceed the tank's capacity is no larger than $5/32$?
- (e) Find the expected value of the daily cost of the fertilizer used by the farmer, as a function of C .

4. **[Exponential distribution]**

Suppose that the duration in minutes of long-distance telephone conversations follows an exponential distribution with parameter $\lambda = 1/2$. Find the probability that the duration of the conversation will:

- (a) exceed 5 minutes?
- (b) be less than 6 minutes?
- (c) be between 5 and 6 minutes?
- (d) be less than 6 minutes, given that it was greater than 5 minutes?

5. **[Uniform and exponential distributions]**

Let X be a continuous random variable uniformly distributed over the interval $(0, 10)$ and let Y be continuous random variable exponentially distributed with parameter $\lambda = 1$. What is the probability that

- (a) the integer part of X will be a 0?
- (b) the first decimal digit of Y will be a 1?
- (c) the first decimal digit of the square root of X will be a 2?
- (d) the second decimal digit of Y will be a 3?

6. **[Uniform and exponential distributions]**

Suppose that a smart phone manufacturer gets the batteries for their phone from two different factories but there is no way to tell which factory each battery comes from, and each battery is equally likely to come from either factory. The lifetime of a battery from one factory is exponentially distributed with parameter $\lambda = 0.2$, in units of $(years)^{-1}$, while the lifetime of a battery from the other factory is uniformly distributed from zero to three years.

- (a) Find the CDF of the lifetime of a randomly chosen radio.
- (b) Suppose you bought a radio and after five years it is still working. What is the conditional probability it will still work for at least four more years?
- (c) Suppose your roommate bought a radio and after one year it is still working. What is the conditional probability it will work for at least four more years?