University of Illinois Spring 2010

ECE 313: Problem Set 3

Discrete Random Variables: pmf, expectation, LOTUS, and variance

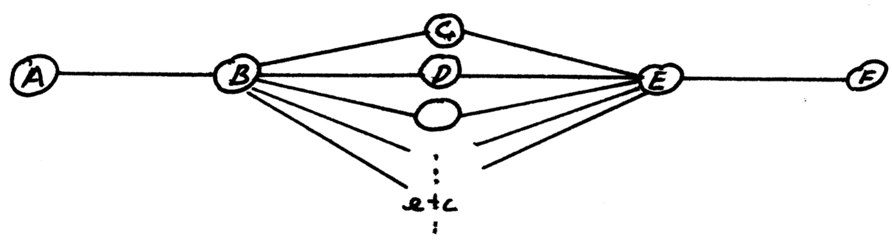
Due:	Wednesday February 10 at 4 p.m.
Reading:	Ross, Chapter 3.4 and 4
Noncredit Exercises:	DO NOT turn these in. Chapter 4: Problems 2, 7, 13, 28, 35, 39, 40-43 Theoretical Exercises 11, 13, 15; Self-Test Problems 1-10.

1. [**Cumulative distribution function**] Page 174, #4.19

2. [**Binomial random variable**] Page 176, #4.43

3. [**Flying used to be fun**] Air Illini overbooks flights to reduce losses due to passengers not showing up for a flight. Suppose that 95 passengers have reservations for a flight with only 92 seats. Passengers make up their minds independently about showing up for the flight. The probability that each passenger will not show up is 0.04. Find the probability that there will be seats available for all passengers who show up.

4. [**Keep the communication channel open**] How many independent microwave links are needed to ensure communication between B and E for at least 99% of the time?



5. [**A little math with a geometric random variable**] The probability mass function for a discrete random variable X is defined to be $P\{X=i\}=C\left(\frac{3}{4}\right)^{i-1}, i=1,2,3,\ldots,\infty$ and C is a constant.
(a) Determine the value of C
(b) Compute $P\{X>8\}$ and $F_X(4)$.

6. [**The mean and variance of the Poisson distribution vs. binomial distribution**] Fix $\lambda > 0$. The Poisson pmf with parameter λ is approximately the same as a binomial pmf with parameters n and p, with n much larger than λ and $p = \lambda/n$. Let $n \to \infty$ with $p = \lambda/n$. Show that the mean and variance of the binomial pmf with parameters n and p converge to the mean and variance of a Poission random variable with parameter λ.

7. [**The cost of flying**] Eight persons have purchased tickets (\$F per person) for travel in a 5-passenger plane on a scheduled airline flight in Randomtania. The number of persons who actually show up to travel can be modeled as a binomial random variable X with parameters (8,

0.5). Naturally, if more than 5 persons show up, only 5 get to go and the rest are left behind. Let Y denote the number of persons left behind.
(a) Determine E[X].
(b) Determine the pmf of Y.
(c) Determine E[Y]. Calculate this in two ways: (i) from your answer to part (b), and (ii) by using the fact that Y is a function of X, and hence LOTUS allows us to calculate E[Y] directly from the pmf of X .
(d) The flight costs the airline $200 plus $10 for each passenger carried on board. (Even if no passengers show up, the flight must still go because the plane is needed at the destination for use in the return flight). According to Randomtania Aviation Administration (RAA) rules, passengers who do not show up are SOL; they cannot use their tickets on another flight and they cannot get a refund either. On the other hand, each bumped passenger gets a full refund of $F plus $20 as compensation for being denied boarding. Let Z denote the net profit to the airline from this flight. Use LOTUS to express E[Z] as a function of F and determine the value of F for which the average profit is exactly 0, i.e. the break-even point.
(e) Determine the pmf of Z