University of Illinois **Spring 2009**

ECE 313: Problem Set 4
Binomial, Geometric, and Poisson Random Variables

Due:	Wednesday February 18 at 4 p.m.
Reading:	*Ross* Sections 4.7-4.10.
Noncredit Exercises:	Chap. 4: Problems 43-85
	Theoretical Exercises 10, 11, 13-20, Self-test problems 11-14

1. [**The Binomial Random Variable I**]
Suppose that eight bits are used to encode a standard ASCII character; the eighth bit is a checksum bit set to the XOR of the seven other bits. Suppose that these bits are transmitted on an optical fiber, and that the probability of an error in any bit transmission is $p = 0.001$. The receiver detects an error whenever the XOR of the bits of a received 8-bit word is non-zero.

   (a) Specify the pmf for the random variable $\mathbf{X}$ that denotes the number of error bits in an eight-bit word.

   (b) Find p_{ue}, the probability that a received eight-bit word will contain undetected errors.

   (c) Suppose n eight-bit words are transmitted. Determine the pmf for the random variable $\mathbf{Y}$ that denotes the number received words that contain undetected errors.

   (d) Compute $E[\mathbf{Y}]$ for $n = 1, 10, 100, 1000, 10000$.

2. [**Geometric Random Variables**]
Consider again a communication system that transmits bits with independent probability of error $p = 0.001$, with each group of eight bits corresponding to a seven-bit ASCII code with checksum.

   (a) Give the pmf for the random variable $\mathbf{Y}$ that denotes the number of bits that are correctly transmitted before the first error occurs. Note: in this problem there is no advantage in grouping the bits into eight-bit words.

   (b) Give the pmf for the random variable $\mathbf{Z}$ that denotes the number of eight-bit words that are transmitted before the first undetected error occurs.

   (c) Suppose a communication company specifies that on average 10KB (i.e., 10×2^{10} bytes) should be transmitted before the first undetected error occurs. Find the maximum value of p that achieves this specification.

3. [**Poisson Random Variables**]
Consider for a third time a communication system that transmits bits with independent probability of error $p = 0.001$, with each group of eight bits corresponding to a seven-bit ASCII code with checksum. Suppose that the number of undetected errors in a transmission of n eight-bit words is modeled as a Poisson random variable $\mathbf{X}$, with parameter $\lambda = np_{\text{ue}}$.

   (a) Compute $E[\mathbf{X}]$ for $n = 1, 10, 100, 1000, 10000$.

   (b) Compare these results to your results from problem 1.

   (c) Compute the probability that a transmission will contain at least one undetected error using both the binomial and the Poisson pmf for $n = 1, 10, 100, 1000, 10000$.

4. **[The Binomial Random Variable II]**
A newsboy purchases 50 newspapers for 35 cents each and sells them for 60 cents each. He can recycle any unsold papers and recoup 25 cents for each. A total of 100 people pass by him each day, and each decides (independently of all other passers by) to ask to buy a paper with probability 0.6.

   (a) What is the probability that the newsboy sells all 50 papers?

   (b) Let Z denote the daily profit (in cents) that the newsboy makes. What is his maximum profit? What is his maximum loss? More generally, express Z in terms of X.

   (c) Compute the newsboy's average daily profit.
   This is not too surprising since there is a better than 98% chance he sells all his papers and makes 1250 every day. (It would have been surprising if the average profit had been more than 1250, the max profit!)

   (d) One day, after the newsboy has sold all of his papers, an angry customer demands a paper. After this experience, the newsboy decides to increase the number of papers he buys. Being fairly cautious, he decides to begin by buying one extra paper.
   What is the probability that he sells this extra paper?
   What is the average *additional* profit that he makes from this paper?

   (e) Encouraged by his success, on the next day he decides to buy 52 papers.
   What is the probability that he sells this extra paper (i.e., the 52nd paper)?
   What is the average *additional* profit that he makes from this 52nd paper? (Note that you are now being asked to compute his average additional profit from the 52nd paper, over and above the profit from 51 papers that he bought the previous day).
   Is the average *additional* profit from the 52nd paper larger or smaller than the average *additional* profit from the 51st paper?

For full credit, correct numerical answers must be given. Feel free to use a computer (e.g., MATLAB, Mathematica) for this problem.

5. **[The Binomial Random Variable III]**
A New Yorker runs an investment management service that has the stated goal of doubling the value of his clients' investments in a week via day trading. His brochure boasts that, "On average, my clients triple their money in five weeks!" After poring over back issues of the *Wall Street Journal* you learn the truth: at the end of any week, the investments of his clients will have doubled with probability 0.5, and will have decreased by 50% with probability 0.5. Thus, at the end of the first week, an initial investment of \$32 will be worth either \$64 or \$16, each with probability 0.5. Performance in any week is independent of performance during the other weeks. Anxious to apply your new skills in probability theory, you decide to invest \$32, and to let that investment ride for five weeks (in fact, you decide not to even look at the stock prices until the five weeks are over). Let the random variable X denote the value in dollars of your investment at the end of a five week period.

   (a) What are the possible values of X?

   (b) What is the pmf of the random variable X?

   (c) What is the expected value of X? Is the TV commercial accurate?

   (d) What is the probability that you will lose money on your investment?