University of Illinois Spring 2009

ECE 313: Hour Exam I

Monday March 2, 2009
7:00 p.m. — 8:00 p.m.
100 Noyes Laboratory

1. [**24 points**] **Remember, answers without appropriate justification will receive very little credit.** An urn contains four balls numbered 1 through 4. Two balls are drawn randomly (without replacement) from the urn and placed in a hat.

   (a) [**7 points**] Find the probability that the 3 ball is in the hat.

   (b) [**10 points**] Define the random variable $\mathbb{X}$ to be the sum of the numbers on the two balls that are in the *hat*. Find the probability mass function (pmf) of $\mathbb{X}$.

   (c) [**7 points**] While your back is turned, a thief steals the 3 ball (regardless or whether it is in the hat or the urn). Now a ball is drawn randomly from the *hat*. Find the probability that this ball has an odd number on it.

2. [**30 points**] Fred and Wilma play a series of games in which they take turns tossing a fair coin. The first one to toss a Head in a game wins that game, and the loser pays the winner \$1. Fred tosses first in the first game.

   (a) [**10 points**] Find P(Fred wins the first game).

   In the games that follow the first game, the *loser of a game gets to toss first in the next game.* Let $\mathbb{X}$ denote the amount of money (in dollars) that Wilma has won after three games. Remember that $\mathbb{X}$ will be negative if Wilma has lost money.

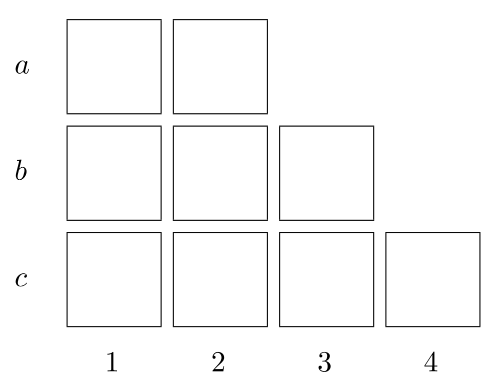
   (b) [**20 points**] Find the probability mass function (pmf) of $\mathbb{X}$.

3. [**16 points**] This problem has two unrelated parts.

   (a) [**8 points**] A and B are two events defined on a sample space Ω. If the probability that *at least one* of the events *occurs* is 0.8 and the probability that *at least one* of the events *does not occur* is also 0.8, what is the probability that *exactly one of the events occurs?*

   (b) [**8 points**] $\mathbb{X}$ denotes a binomial random variable with parameters $(100, 0.2)$.
   Find the value of $\mathsf{E}[\mathbb{X}^2]$.

4. [**30 points**] Nine squares are arranged in three rows and four columns, labeled as shown.



One of the squares is selected by the following two-step procedure. First, one of the rows, a, b, or c, is chosen with each row being equally likely to be picked. Then, one square is chosen from the selected row, with each square in the row being equally likely to be picked.

(a) **[9 points]** Within each square *shown above*, write the probability that that square is selected. (Hint: Make sure the sum over all nine squares is one.)

For $i \in \{1, 2, 3\}$, let R_i be the event that the square selected is in the i-th *row*.
For $j \in \{1, 2, 3, 4\}$, let C_j be the event that the square selected is in the j-th *column*..

(b) **[7 points]** Find $P(C_2)$.

(c) **[7 points]** Find $P(R_1 \mid C_2)$.

(d) **[7 points]** Find $P(C_2 \mid R_2)$.