ECE 313: Hour Exam I

1. [30 points] Let \( A, B, \) and \( C \) denote three events defined on a sample space \( \Omega \), and suppose that \( P(A) = 0.5, P(B) = 0.4, P(C) = 0.3, P(B^c \cap C) = 0.2, \) and \( P(A \cap B^c \cap C^c) = 0.1 \).

Find the following probabilities: \( P(B \cap C), P(B \cap C^c), P((A \cup B \cup C)^c), P(A^c \cap B), \) and \( P(B^c \cap C^c) \).

If any probability cannot be computed from the given data, check the corresponding box and leave the answer area blank.

2. (a) [10 points] If \( X \) is a Poisson random variable with mean 4, what is \( \text{var}(2 + 3X) \)?
(b) [10 points] Let \( Y \) be a negative binomial random variable with parameters \((n, p)\) where \( n \) is known, but the value of \( p \) is unknown. It is observed that \( \{Y = k\} \). What is the maximum-likelihood estimate \( \hat{p} \) of the parameter \( p \)?

3. (a) [8 points] Dilbert has 3 coins in his pocket, 2 of which are fair coins while the third is a biased coin with \( P(H) = p \neq \frac{1}{2} \). The probability that a coin chosen at random from his pocket will land Tails is \( \frac{7}{12} \). What is the value of \( p \)?
(b) [18 points] Let \( A \) and \( B \) denote events defined on a sample space. Given that \( P(A) = \frac{3}{5}, P(B) = \frac{2}{5}, \) and \( P(B|A) = \frac{1}{3}, \) find \( P(A|B), P(A^c \cup B^c) \) and \( P(B^c|A^c) \).

4. [24 points] At the Democratic National Convention, Hillary Clinton and Barack Obama have equal numbers of delegates committed to them, and neither candidate can win the nomination on a ballot. In desperation, the Convention decides that the two candidates shall debate each other and the winner shall be the nominee of the Democratic Party. On a debate, Clinton wins (event \( H \)) with probability \( P(H) \), and Obama wins (event \( B \)) with probability \( P(B) \). A draw (event \( D \)) occurs (that is, neither wins) with probability \( P(D) = 1 - P(H) - P(B) > 0 \). In case of a draw, another debate is held. Successive debates can be regarded as independent trials, and continue until either event \( H \) or \( B \) occurs, and the Democratic nominee is chosen.

Express the answers to the following questions in terms of \( P(H) \) and \( P(B) \).

(a) [8 points] What is the probability that Hillary is the Democratic nominee?

(b) [8 points] Given that no more than 5 debates were held, what is the conditional probability that Obama won the nomination?

(c) [8 points] What is the expected number of debates at the Democratic National Convention?