University of Illinois at Urbana-Champaign

ECE 413: Probability with Engineering Applications

Spring 2007
Exam I

Monday, February 26, 2007

Name: ____

- You have 60 minutes for this exam. The exam is closed book and closed note, except you may consult both sides of one $8.5'' \times 11''$ sheet of notes in ten point font size or larger, or equivalent handwriting size.
- Calculators, laptop computers, Palm Pilots, two-way e-mail pagers, etc. may not be used.
- Write your answers in the spaces provided.
- **Please show all of your work. Answers without appropriate justification will receive very little credit.** If you need extra space, use the back of the previous page.

Score:

1. __________ (36 pts.)

2. __________ (16 pts.)

3. __________ (12 pts.)

4. __________ (14 pts.)

5. __________ (22 pts.)

Total: ________(100 pts.)

Problem 1 *(36 points)* An experiment consists of rolling three fair dice. The rolls are independent trials. Let A be the event that the numbers showing on the three dice are all even, and let B be the event that the numbers showing on the three dice are different from each other.

(a) Express the set of possible outcomes Ω using mathematical set notation.

(b) Find $P(A)$.

(c) Find $P(B)$.

(d) Find $P(AB)$.

(e) Find $P(A \cup B)$.

(f) *Sketch and carefully label* the probability mass function of X, where X denotes the number of distinct numbers showing on the dice.

Problem 2 *(16 points)* Let X be a random variable with mean 4 and variance 16.

(a) Find the numerical value of $E[X^2]$. Remember to explain your reasoning.

(b) Find the numerical value of $E[(X+2)(X+3)]$. Show your work.

Problem 3 *(12 points)* Suppose that a random variable X has the pmf

$$p_X(k) = \begin{cases} (k-1)p^2(1-p)^{k-2} & k = 2, 3, \ldots \\ 0 & \text{else} \end{cases}$$

where p is an unknown parameter with $0 < p < 1$. (i.e., X has the negative binomial distribution with parameters p and $r = 2$.) Suppose it is observed that $X = 14$. What is the maximum likelihood estimate of p? Show your work!

Problem 4 *(14 points)* Let A and B denote events such that $P(A) = 0.6, P(B) = 0.5$, and $P(A \,|\, B) = 0.4$.

(a) Find $P(A \cup B)$.

(b) Find $P(B^c \,|\, A^c)$.

Problem 5 *(22 points)* Consider repeated independent tosses of a biased coin with $P(\text{Heads}) = p$, and let X denote the number of tosses required to observe both one Head and one Tail.

(a) What is the minimum possible value of X?

(b) Find the probability mass function of X.

(c) Find the expected value of X. (Find a closed form answer, with no infinite sum.)