University of Illinois Spring 2005

ECE 413: Problem Set 6

Due:	Wednesday March 2 at the beginning of class.
Reminder:	Hour Exam I is on Monday February 28, 7 p.m. in 269 Everitt Lab.
Reading:	Ross, Chapter 3
Noncredit Exercises:	**Chapter 3:** 53, 58, 59, 62, 63, 70-74, 78, 81

This Problem Set contains six problems

1. In Problem 5 of Problem Set 2, we found that the House of Commons Search Committee randomly selects one of 20 short lists of three candidates from the set {Andy, Beth, Chuck, Di, Eddie, Fergie}. Before the House of Commons selects the new monarch, however, Di passes away. Now, due to partisan politics, however, the House will always choose Beth as monarch if she is on the short list If Beth is not on the short list, then the House chooses at random from the three (only two if the list originally included Di) on the list.

(**a**) What is the probability that Beth is chosen as the monarch?

(**b**) What is the probability that Chuck is chosen as the monarch?

(**c**) Given that Beth was chosen as the monarch, what is the conditional probability that Di was originally on the short list? that Chuck was on the short list?

2. Let $\mathcal{X}$ denote a negative binomial (or Pascal) random variable with parameters (r, p). Then, $\mathcal{X}$ counts the number of trials required to observe r successes where the probability of success on any trial is p. Given that $\mathcal{X} = n$, what is the conditional probability that the i-th trial resulted in a success? To avoid trivialities, assume that $n > r$ and also that $n > i$.

3. ["Take me out to the ball game...] A baseball pitcher's repertoire is limited to *fastballs* (event F), *curve balls* (event C) and *sliders* (event S). It is known that $P(C) = 2P(F)$. Whether the event H that the batter hits the ball occurs depends on the pitch, and it is known that $P(H|F) = 2/5, P(H|C) = 1/4$, and $P(H|S) = 1/6$. If $P(H) = 1/4$, what is $P(C)$?

4. Monty Hall, the host of the TV game show "Let's Make A Deal" shows you three curtains. One curtain conceals a car, while the other two conceal goats. All three curtains are equally likely to conceal the car. He offers you the following "deal": pick a curtain, and you can have whatever is behind it. When you pick a curtain, instead of giving you your just deserts, Monty (who knows where the car is) opens one of the remaining curtains to show you that there is a goat behind it, and offers the following "new, improved deal" : you can either stick with your original choice, or switch to the remaining (unopened) curtain. Amidst the deafening roars of "Stand pat" and "Switch, you idiot" from the crowd, Monty points out that previously your chances of winning were 1/3. Now, since you know that the car is behind one of the two unopened curtains, your chances of winning have increased to 1/2, and thus the new improved deal is indeed better. Use the theorem of total probability to determine

(**a**) the probability of winning if you always switch.

(**b**) the probability of winning if you would rather fight than switch.

(**c**) whether Monty is correct in asserting that if you choose randomly betweeen the two unopened curtains, you have a probability of winning of 1/2.

(**d**) Having disposed of your goat, you return the next day to the show, and this time, Monty calls you *and* your friend to come on down and choose one curtain each. Which is better: to be the first to pick a curtain or the second? Or does it not make a difference? This time, Monty opens the curtain chosen by your friend to reveal a goat and sends him back to his seat. He now asks whether you want to stick with your original choice or switch to the the third (unchosen) curtain. Which choice gives you a larger chance of winning the car?

Note: Everybody knows that the rules of the game of parts (a)-(c) are that Monty always opens one of the two unchosen curtains and he always offers the "new improved deal," i.e. he never opens a curtain to reveal the prize (saying "Oops, you lose; return to your seat."). In the game of part (d), he always opens one of the chosen curtains to eliminate one of the contestants and then always offers the other contestant the chance to switch.

5. At the County Fair, you see a man sitting at a table and rapidly rolling a pea between three walnut shells. "Step right up, me bucko, and try your luck! The hand is quicker than the eye!" he says, and hides the pea under one of the shells. You have no idea which shell is covering the pea, but you point to one shell at random and bet that the pea is under it. The man picks up one of the shells that you didn't choose, and shows you that the pea is not underneath that shell. He asks if you would like to switch your bet to the other unchosen shell. Should you accept the offer? Why or why not? How does this game differ from the one analyzed in Problem 4 parts (a)-(c)?

6. The ToyAuto Company needs to decide which of the following two methods provides more reliable transportation:

- a single gigantic car with N engines, N transmissions, N brakes, ... etc. that works (i.e. provides us with transportation) as long as *at least one* of its engines and *at least one* of its transmissions, and *at least one* of its brakes ... works.
- N separate ordinary cars that fail as soon as any one of their parts fail, but which together provide us with transportation as long as at least one car is in working condition.

Each car is made of M different types of parts, and (at least) one part of each different type must work for the car to work. Each part fails with probability p and all the failures are independent events.

(**a**) For each method, find the probability of system failure (we have no transportation!) in terms of p, N and M

(**b**) Suppose that $M = 5$ and $p = 0.2$. If it is desired that the system failure probability be less than 0.001, what should N be with each method?

(**c**) Repeat part (b) assuming that $M = 1000$.