University of Illinois Spring 2005

ECE 413: Hour Exam II
Monday April 11, 2005
7:00 p.m. — 8:00 p.m.

1. **[20 points]** An ECE 413 student seeks predictions from three psychics A, B. and C as to whether the student will pass the course. The psychics predict that the student will pass with probabilities $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$, and $P(C) = \frac{3}{4}$ respectively,

   Assume that A, B, and C are *mutually independent* events, and let D denote the event that at least two of the three psychics predict that the student will pass.

   (a) **[10 points]**What is $P(D)$?

   (b) **[10 points]** Given that event D occurred, what is the probability that the most pessimistic psychic A predicted that the student will *fail* ECE 413?

2. **[30 points]** A photodetector counts photons for 10 nanoseconds to decide if a distant light source is emitting light. When the light source is *not* emitting light, some photons are still counted by the detector due to the ambient background radiation.

   The number of photons counted in 10 nanoseconds is modeled as a Poisson random variable $\mathcal{X}$ whose parameter λ has value $\ln(9)$ if the light source *is not* emitting light (hypothesis H_0), and value $\ln(27)$ if the source *is* emitting light (hypothesis H_1). The maximum-likelihood detector decides that H_1 is the true hypothesis if and only if the *likelihood ratio* $\Lambda(u) = p_1(u)/p_0(u)$ exceeds 1.

   (a) **[10 points]** What is the value of $\Lambda(k)$ when k photons have been counted?

   (b) **[10 points]** What value(s) of $\mathcal{X}$ result in a decision in favor of hypothesis H_1?

   (c) **[10 points]** Express the *false alarm* probability P_{FA} and the *missed detection* or *false dismissal* probability P_{MD} of the maximum-likelihood decision rule as *finite* series, that is, in a form that can be readily evaluated by a scientist or engineer who is armed with a calculator, but does not know probability jargon. DO NOT use your calculator: numerical values are *not required*, only the formula.

3. **[36 points]** $\mathcal{X}$ denotes a *uniform* random variable with mean 1 and variance 3.

   (a) **[12 points]** Find $P\{\mathcal{X} < 0\}$.

   (b) **[12 points]** Find $\mathsf{E}[|\mathcal{X}|]$.

   (c) **[12 points]** Find the pdf of $\mathcal{Y} = |\mathcal{X}|$. In order to receive full credit, you must specify the value of $f_{\mathcal{Y}}(v)$ for all $v, -\infty < v < \infty$.

4. **[14 points]** $\mathcal{X}$ is a Gaussian random variable with mean 2 and variance 25.

   Find $P\{|\mathcal{X} - 4| > 3\}$ and $P\{\mathcal{X} < 3 | \mathcal{X} > 2\}$ using the table of values of the unit Gaussian CDF $\Phi(\cdot)$ on the last page of this exam booklet.