University **Problem Set #3** **ECE 313**
of Illinois **Spring 2002**

Assigned: Wednesday, January 30
Due: Wednesday, February 6
Reading: Ross, Chapters 2.1–2.5, 4.1, 4.3–4.5, 4 .7
Noncredit Exercises: (Do not turn these in) Chapter 4, Problems: 2, 7, 13, 28, 35, 39, 40-43; Theoretical Exercises: 11, 13, 15.

Problems:

1. Let A, B, and C denote three events defined on a sample space Ω, and suppose that

$P(A) = 0.3$, $P(B) = 0.3$, $P(C) = 0.5$, and $P(A \cap B^c) = P(A^c \cap B^c \cap C) = 0.2$.

Find $P(A \cap B)$, $P(A^c \cap B)$, $P((A \quad B \quad C)^c)$, and $P(B \cap C^c)$.
If any of these probabilities cannot be computed from the given information, find upper and lower bounds (preferably other than 1 and 0 !!) on these probabilities.

2. The experiment consists of picking a student from the set of all UIUC students registered this semester. It is **not** necessary to assume that all students are equally likely to be picked, but you may make this assumption if it makes you feel happier and more confident.

(a) Let A and B denote the events that the student picked has had respectively four years of science (FYS) and calculus in high school. Let $P(A) = 0.45$ and $P(B) = 0.35$. If the probability that the student had neither FYS nor calculus is 0.3, what is the probability that the student had both FYS **and** calculus? What is the probability that the student had FYS but **not** calculus ?

(b) Let C denote the event that the student is registered in ECE 313, and let A and B be as in part (a). Suppose that $P(A \cap B \cap C)=0.002$. What is the probability that the student picked is not registered in ECE 313, but did have both FYS **and** calculus ? If the probability that the student picked is registered in ECE 313, and has had either FYS or calculus (but not both) is 0.004, and if students who had neither FYS nor calculus did not register in ECE 313, what is P(C)?

(c) Using the data given in parts (a) and (b), which of the following probabilities can you compute? It is not necessary to actually compute each probability.

$P(A \quad C)$, $P(A \quad B \quad C)$, $P(A \quad B \quad C^c)$, $P(A^cB^cC^c)$, $P(A^cBC^c)$, $P(ABC^c)$

3. Let A, B, C denote three events defined on a sample space. Show that

$$\frac{P(A) + P(B) + P(C)}{3} \leq P(A \quad B \quad C) \leq P(A) + P(B) + P(C)$$

4. Find $P(A \quad (B^c \quad C^c)^c)$ in each of the following four cases:

(a) A, B, and C are mutually exclusive events and $P(A) = 1/3$.
(b) $P(A) = 2P(BC) = 4P(ABC) = 1/2$.
(c) $P(A) = 1/2$, $P(BC) = 1/3$, and $P(AC) = 0$.
(d) $P(A^c \cap (B^c \quad C^c)) = 0.6$.

5. Eight people hold reservations for travel in a 5-passenger limousine from Champaign to St. Louis. The number of persons who actually show up to travel can be modeled as a binomial random variable **X** with parameters $(8,\frac{1}{2})$. Note: $2^8 = 256$. If more than 5 people show up, only the first 5 get to go, and the rest are left behind. What is the average number of passengers who are left behind?

6. Use a spreadsheet/Mathematica/MATLAB for this problem.
Let A denote an event of probability p.

(a) For p = 0.1, 0.25. 0.4, 0.5, 0.6, 0.75, and 0.9, find the numerical values of the probabilities that A occurs 0, 1, 2, … , 10 times on 10 trials of the experiment.

(b) You have, in effect, computed the probability mass function for a binomial random variable **X** with parameters (10, p) for seven choices of p. For each value of p, draw a bar graph of the pmf. (For p = 0.5, the answer is on page 151 (5th ed.) or 146 (6th ed.) of Ross!)

(c) What is the relationship between the pmfs for the cases p = 0.1 and p = 0.9? for the cases p = 0.25 and p = 0.75? for the cases p = 0.4 and 0.6?

(d) Prove mathematically that if **X** is a binomial random variable with parameters (n, p), then **Y** = n – **X** is a binomial random variable with parameters (n, 1–p).

(e) From each of the seven graphs of part (b), find the value of k for which P{**X** = k} is maximum. Compare your results to the prediction of Proposition 6.1, p. 145, of Ross.

7. [“Eat your broccoli, dear. It’s good for you”] Let A, B, and C denote the events that your mother serves respectively asparagus, broccoli, and cauliflower for dinner. From (bitter?) experience you know that these events are mutually exclusive (i.e. you get only one vegetable each day) and that P(A) = 0.2, P(B) = 0.5, and P(C) = 0.3. Each day is an independent trial: that is, your mother, a lady of formidable temperament albeit limited culinary skills, makes *independent* decisions (e.g. without taking into account your opinion that Cheetos is a vegetable suitable for serving with any entree) about the vegetable to serve each day. Over a three day period, what is the probability that

(a) she serves the same vegetable on all three days ?

(b) she serves the same vegetable exactly two days out of three ?

(c) she serves different vegetables on the three days ?