Assigned: Wednesday, January 16
Due: Wednesday, January 23
Reading: Ross, Chapter 1.1–1.5, Chapter 2.1–2.5 and 2.7
Noncredit Exercises: (Do not turn these in)

	Problems	Theoretical Exercises	Self-Test Problems
Chapter 1	1–5, 7, 9	4, 8, 13	1-15
Chapter 2	3, 4, 9, 10, 11-14	1–3, 6, 7, 10, 11, 12, 16, 19, 20	1-8

Problems: These problems are based entirely on material covered in the *prerequisites* to this course. You should have mastered this stuff already, but may need to review the material one more time before starting the course. Think of this problem set as a diagnostic aid: if you cannot solve *all* these problems correctly, you will have difficulty in comprehending the material in the latter half of this course. It is not in your best interest to discover *after* the drop date that you really don't understand calculus as well as you thought you did, and that consequently you are in some danger of failing this course.

Do not use Mathematica or Matlab or a calculator etc. to do these problems except when you are specifically asked to do so.

1.(a) Without using your calculator, determine whether the following statement is true.

$$\exp(-1) = e^{-1} < 1$$

Explain your reasoning.

(b) Now use your calculator to compute $\exp(-1)$, $\exp(-1^2)$, and $\exp(0-1^2)$. According to your calculator, do all three expressions have the same value?

(c) Now use the Microsoft spreadsheet program Excel to compute these quantities. (Enter the three formulas =EXP(–1), =EXP(–1^2), and =EXP(0–1^2) into different cells in the spreadsheet. The PASTE FUNCTION command may be needed to get the EXP inserted)

(d) Without using a calculator, determine whether 111111111^2 equals 12345678987654321.

(e) True or False? The solutions to the quadratic equation $ax^2 + bx + c = 0$ are given by $\dfrac{2c}{-b \pm \sqrt{b^2 - 4ac}}$. Here, to avoid trivialities, assume that a, b, and c are all nonzero.

2. The angles in this problem are expressed in **degrees** and **not in** the **radians** more commonly used in mathematical circles.

(a) Use your calculator to evaluate $\cot(10^\circ)\cot(30^\circ)\cot(50^\circ)\cot(70^\circ)$ *without* ***writing*** *down intermediate results such as the values of* $\cot(10^\circ)$, $\cot(30^\circ)$, *etc and re-entering the numbers into your calculator.* If your calculator cannot be used in this fashion, you are urged to replace it with a more sophisticated machine.

(b) If your calculator's arithmetic unit is designed in accordance with the IEEE Standard for floating-point arithmetic, you should have obtained exactly 3 as the answer to part (a). Does it surprise you that $\cot(10^\circ)\cot(30^\circ)\cot(50^\circ)\cot(70^\circ) = 3$?
Find four integers a, b, c, and d such that $0 < a < b < c < d < 90$ and $\cot(a^\circ)\cot(b^\circ)\cot(c^\circ)\cot(d^\circ)$ is a rational number other than 3 or 1.

(c) Find the value of $\dfrac{1}{\sqrt{2}+\sqrt{1}} + \dfrac{1}{\sqrt{3}+\sqrt{2}} + \dfrac{1}{\sqrt{4}+\sqrt{3}} + \ldots + \dfrac{1}{\sqrt{100}+\sqrt{99}}$ You can use your programmable calculator if you wish.

3. In **this** problem, all **angles** are expressed in **radians**.

(a) Use your calculator to evaluate $\sqrt{52}\cos(3^{-1}\arctan(18\sqrt{3}/35))$.

(b) In this part, you have to find the limit of $\dfrac{1}{[\sin x]^2} - \dfrac{1}{x^2}$ as x approaches 0. Use your calculator to evaluate this function for *small* values of x say, $x = 10^{-1}$, $x = 10^{-2}$, $x = 10^{-3}$, etc. Does the function seem to be approaching a limit, and if so, what do you think is the limit? Now, use what you have learned about limits in calculus to find

$\lim_{x \to 0} \frac{1}{[\sin x]^2} - \frac{1}{x^2}$ analytically. (Hint: the answer is not 0, or 1, or ∞)

(c) Find the maxima of $f(x) = x^{25}(1.00001)^{-x}$ for $x > 0$. (If you have a graphing calculator, try it on this problem; otherwise just use standard calculus methods)

4.(a) What is the value of $\int_{-2}^{1} |x|\, dx$? the value of $\int_{-2}^{1} x(1-x)^{19}\, dx$?

(b) Prove or disprove: there exists a function f(x) satisfying **both** of the following two conditions:

(i) $f(x) \geq 0$ for all real numbers x in the range $-2 \leq x \leq 1$,

(ii) $\int_{-2}^{1} f(x)dx < 0$. (Hint: Does either function of part (a) satisfy both conditions?)

(c) Let $\frac{d}{dx}f(x) = g(x)$ for $-\infty < x < \infty$. Which of the following statements are true for all x, $-\infty < x < \infty$? In parts (iv)-(vi), C denotes an arbitrary constant.

(i) $\frac{d}{dx}f(-x) = g(-x)$. (ii) $\frac{d}{dx}f(x^2) = 2x\, g(x^2)$. (iii) $\frac{d}{dx}\exp(f(x^2)) = \exp(f(x^2))\, g(x^2)$.

(iv) $\int g(-x)dx = -f(-x) + C$. (v) $\int g(x^2)dx = f(x^2)/(2x) + C$. (vi) $\int \frac{g(x)}{f(x)}dx = \ln(f(x)) + C$.

(d) Evaluate $\int_{0}^{\infty} x \bullet \exp(-x^2/2)dx$

5.(a) What is the derivative of arctan(x)? (You can look up the answer if you like!)

(b) I denotes the value of the integral $\int_{-1}^{1} \frac{2}{1+x^2}\, dx$. Use the result of part (a) to show that I = π.

(c) J denotes the value of the integral $\int_{-1}^{1} \frac{2}{1+y^2}\, dy$. State True or False: I equals J.

(d) Make the substitution y = 1/x in the integral of part (b) and simplify the integrand.

Do you get the result that $I = \int_{-1}^{1} \frac{2}{1+x^2}\, dx = \int_{-1}^{1} \frac{-2}{1+y^2}\, dy = -J$?

If so, does this contradict your answer to part (c)?

(e) I can equal **both** J **and** –J if and only if I = J = 0. Since you showed in part (b) that I = π, does this mean π = 0? (Such a result would greatly simplify a **lot** of engineering math!)

6.(a) Integrate $f(x, y) = \begin{cases} 6, & 0 < y < x,\ 0 < x < 1, \\ 0, & \text{elsewhere}, \end{cases}$ over the region $\{(x,y) : y > x^2\}$.

(b) Compute the integral of $(x^2 + y^2)^{-2}$ over the region $\{(x,y) : x^2+y^2 > 2\}$.