1. Let A, B, and C respectively denote the events that the U-O, O-C, and U-C links are in working condition. The capacities are marked on the Karnaugh map below (in which each cell has probability 1/8). It is easily seen that \( Z \) takes on values 0, 56, 100, and 156 with probabilities 3/8, 1/8, 3/8, and 1/8 respectively, and hence \( E[Z] = \frac{0 \times 3}{8} + \frac{56 \times 1}{8} + \frac{100 \times 3}{8} + \frac{156 \times 1}{8} = \frac{512}{8} = 64 \).

2. \( 1 = \int_{-\infty}^{\infty} f_X(u) \, du = \int_{0}^{1} a + bu \, du = au + bu^2/2 \bigg|_{0}^{1} = a + b/2. \)

Also, \( \frac{2}{3} = E[X] = \int_{-\infty}^{\infty} uf_X(u) \, du = \int_{0}^{1} au + bu^2 \, du = au^2/2 + bu^3/3 \bigg|_{0}^{1} = a/2 + b/3. \) Hence, \( a = 0, b = 2. \) The pdf is thus \( 2u \) on \([0, 1]\). \( P\{X < 1/2\} = \int_{0}^{1/2} 2u \, du = 1/4. \) This is even easier if you sketch the pdf as shown.

3. (a) The pdf of \( X \) has value 0.2 for \(-1 \leq u \leq 4\), is as shown below in the left-hand figure below. We have that \( E[Y] = E[|X-2|] = \int_{-1}^{2} 0.2(2-u) \, du + \int_{2}^{4} 0.2(u-2) \, du = 0.2(2u - u^2/2) \bigg|_{-1}^{2} + 0.2(u^2/2 - 2u) \bigg|_{2}^{4} = 1.3. \)

(b) \( Y \) takes on values in the range \([0, 3]\). From the middle and right-hand figures above, we see that for any \( v, 0 \leq v \leq 2, F_Y(v) = P\{Y \leq v\} = P\{2-v \leq X \leq 2+v\} = 0.2(2-v) \), while for any \( v, 2 \leq v \leq 3, F_Y(v) = P\{2-v \leq Y \leq 4\} = 0.2(4-(2-v)) = 0.2(2+v). \) Hence, \( f_Y(v) = \begin{cases} \frac{4}{9}, & 0 \leq v \leq 2, \\ \frac{0.2}{9}, & 2 \leq v \leq 3 \\ 0, & \text{elsewhere} \end{cases} \) which is easily verified to be a valid pdf.

4. (a), (b) As studied in class, \( X \) is an exponential random variable with parameter \( \mu \). Hence, \( P\{X > \tau\} = \exp(-\mu \tau) \) and \( f_X(u) = \mu \exp(-\mu u) \) for \( u > 0 \), and 0 otherwise.

(c) \( P\{X > \tau \mid A\} \) is obviously 0 if \( \tau \geq 3 \), while for \( 0 \leq \tau < 3, 
\begin{align*}
P\{X > \tau \mid A\} &= P\{\text{No arrivals in } (0,\tau) \cap \text{two arrivals in } (0.3)\} \\
&= P\{\text{No arrivals in } (0,\tau) \cap \text{two arrivals in } (0,3)\}/P\{\text{two arrivals in } (0,3)\} \\
&= (\exp(-\mu \tau)\exp(-\mu)(\mu^2)/2!)/\exp(-3\mu)(3\mu^2)/2! = (3-\tau)^2/9. \\
\end{align*} \)

(d) \( f_{X\mid A}(u\mid A) = -\frac{d}{d\tau} P\{X > \tau \mid A\} = (6 - 2\tau)/9, \) 0 \leq \tau < 3, and 0 otherwise.

5. \( P\{|X-4| > 3\} = P\{X > 7\} + P\{X < 1\} = 1 - \Phi\left(\frac{7-2}{5}\right) + \Phi\left(\frac{1-2}{5}\right) = 1 - \Phi(1) + \Phi(-0.2) = 1 - \Phi(1) + \Phi(0.2) = 2 - 0.8413 - 0.5793 = 0.5794. \)

\( P\{X < 3 \mid X > 2\} = P\{2 < X < 3\}/P\{X > 2\} = 2 \frac{\Phi\left(\frac{3-2}{5}\right) - \Phi\left(-\frac{2-2}{5}\right)}{\Phi(0.2) - 0.5} = 1.1586 - 1 = 0.1586 \)