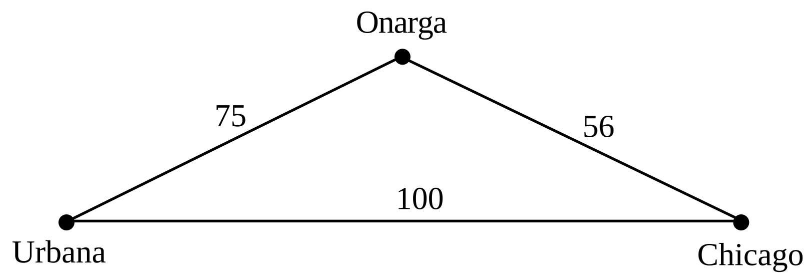
**University of Illinois** | **Second Hour Exam** | **ECE 313 Spring 2002**

**1.** **(20 points)** In the network shown below, the links are telephone trunk lines that have call-carrying capacities as marked. Each link fails with probability $\frac{1}{2}$ independently of the other links, and when it fails, its call-carrying capacity is reduced to 0. Let **Z** denote the call-carrying capacity of this network between Urbana and Chicago. Find the expected value of **Z**.

Remember that you are being asked about the *call-carrying* ***capacity*** of the system, that is, the **maximum** number of calls that **can** be carried, and **not** about the number of calls that **are** being carried. Also, assume that no calls originate or terminate in Onarga and thus the entire capacity of all the trunk lines is available for carrying the Urbana-Chicago traffic.




**2. (20 points)** **X** is a continuous random variable with probability density function

$$f_{\mathbf{X}}(u) = \begin{cases} a + bu, & 0 < u < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

If $E[\mathbf{X}] = \frac{2}{3}$, find $P\{\mathbf{X} < \frac{1}{2}\}$.

If the answer cannot be determined from the given information, check the square ☐ on the left side of the answer box and leave the right side of the answer box blank.

**3.** **(20 points)** Let **X** denote a random variable uniformly distributed on the interval [–1, 4], and let $\mathbf{Y} = |\mathbf{X}-2|$

**(a)** **(6 points)** Find E[**Y**].

**(b)** **(14 points)** Find $f_{\mathbf{Y}}(v)$, the probability density function of **Y.**

**4.** **(25 points)** Consider a Poisson process with arrival rate μ. Let **X** denote the time of the first arrival after t = 0, and let τ denote a nonnegative real number.

If you have the answers to parts (a) and (b) written down on your sheet of notes, you need not show any work for parts (a) and (b). Just *write down* the answer ***and state*** that it is from your sheet of notes.

**(a)** **(5 points)** What is $P\{\mathbf{X} > \tau\}$?

**(b)** **(4 points)** What is the probability density function of **X**?

**(c)** **(10 points)** A is the event that there are exactly two arrivals in the time interval (0, 3]. What is the conditional probability that $\{\mathbf{X} > \tau\}$ *given* that the event A occurred? Be sure to give the answer for all nonnegative values of τ.

**(d)** Recall that **X** is the time of the first arrival after t = 0 in a Poisson process of intensity μ and that A is the event that there are exactly two arrivals in the time interval (0, 3].

**(6 points)** What is $f_{\mathbf{X}|A}(u|A)$, the conditional probability density function of **X** given that A has occurred?

**5.** **(15 points)** **X** denotes a Gaussian random variable with mean 2 and variance 25.

Find $P\{|\mathbf{X} - 4| > 3\}$ and $P\{\mathbf{X} < 3 \mid \mathbf{X} > 2\}$ using the table of values of the unit Gaussian CDF Φ(•) on the last page of this exam booklet.